ENGINEERING PHYSICS

COURSE OBJECTIVES

□ To make the students effectively to achieve an understanding of mechanics.

□ To enable the students to gain knowledge of electromagnetic waves and its applications.

□ To introduce the basics of oscillations, optics and lasers.

□ Equipping the students to be successfully understand the importance of quantum physics.

□ To motivate the students towards the applications of quantum mechanics.

UNIT I MECHANICS

Multi-particle dynamics: Center of mass (CM) – CM of continuous bodies – motion of the CM – kinetic energy of system of particles. Rotation of rigid bodies: Rotational kinematics – rotational kinetic energy and moment of inertia – theorems of M .I –moment of inertia of continuous bodies– M.I of a diatomic molecule – torque – rotational dynamics of rigid bodies – conservation of angular momentum – rotational energy state of a rigid diatomic molecule – gyroscope – torsional pendulum – double pendulum – Introduction to nonlinear oscillations.

UNIT II ELECTROMAGNETIC WAVES PH3151 Syllabus ENGINEERING PHYSICS

The Maxwell's equations – wave equation; Plane electromagnetic waves in vacuum, Conditions on the wave field – properties of electromagnetic waves: speed, amplitude, phase, orientation and waves in matter – polarization – Producing electromagnetic waves – Energy and momentum in EM waves: Intensity, waves from localized sources, momentum and radiation pressure – Cell-phone reception. Reflection and transmission of electromagnetic waves from a non-conducting medium vacuum interface for normal incidence.

UNIT III OSCILLATIONS, OPTICS AND LASERS

Simple harmonic motion – resonance –analogy between electrical and mechanical oscillating systems – waves on a string – standing waves – traveling waves – Energy transfer of a wave – sound waves – Doppler effect. Reflection and refraction of light waves – total internal reflection – interference –Michelson interferometer –Theory of air wedge and experiment.Theory of laser – characteristics – Spontaneous and stimulated emission – Einstein's coefficients – population inversion – Nd-YAG laser, CO_2 laser, semiconductor laser –Basic applications of lasers in industry.

UNIT IV BASIC QUANTUM MECHANICS

Photons and light waves – Electrons and matter waves –Compton effect – The Schrodinger equation (Time dependent and time independent forms) – meaning of wave function – Normalization –Free particle – particle in a infinite potential well: 1D,2D and 3D Boxes- Normalization, probabilities and the correspondence principle.

UNIT V APPLIED QUANTUM MECHANICS PH3151 Syllabus ENGINEERING PHYSICS

The harmonic oscillator(qualitative)- Barrier penetration and quantum tunneling(qualitative)- Tunneling microscope – Resonant diode – Finite potential wells (qualitative)- Bloch's theorem for particles in a periodic potential –Basics of Kronig-Penney model and origin of energy bands.

Unit - 1

Mechanics

1.1 Multiparticle Dynamics:

A mechanical system consists of two or more particles is called multiparticle system.

Let us consider two particles of mass m_1 and m_2 moving in one-dimension with co-ordinates x_1 and x_2.

$\vec{F}_1$ is the force acting on body m_1, then according to Newton's Law,

$$(\text{or})\ m_1 \vec{a}_1 = \vec{F}_1$$

$$m_1 \frac{d^2x_1}{dt^2} = F_1 \quad (\text{or}) \quad m_1\ddot{x}_1 = F_1 \quad \ldots (1) \qquad \left(\because \vec{a} = \frac{d^2\vec{x}_1}{dt^2}\right)$$

where $\frac{d^2x_1}{dt^2}$ is denoted as $\ddot{x}_1$

The force $\vec{F}_1$ can be divided into two parts like,

$$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{1e} \quad \ldots (2)$$

eqn (1) becomes,

$$m_2\ddot{x}_2 = \vec{F}_{12} + \vec{F}_{1e} \quad \ldots (3)$$

m^{ly}, for particle 2,

$$m_2\vec{a}_2 = \vec{F}_2$$

$$m_2\frac{d^2x_2}{dt^2} = m_2\ddot{x}_2 = \vec{F}_2 = \vec{F}_{21} + \vec{F}_{2e} \quad \ldots (4)$$

Total force on system of the two particle is given by adding eqn (3) & (4) we get.

(3)

$$m_1\ddot{x}_1 + m_2\ddot{x}_2 = \vec{F}_{12} + \vec{F}_{21} + \vec{F}_{1e} + \vec{F}_{2e} \quad \cdots (5)$$

By Newton's third law of motion $\vec{F}_{12} = -\vec{F}_{21}$

$$m_1\ddot{x}_1 + m_2\ddot{x}_2 = \vec{F}_{12} + (-\vec{F}_{12}) + \vec{F}_{1e} + \vec{F}_{2e} \quad \cdots (6)$$

$$m_1\ddot{x}_1 + m_2\ddot{x}_2 = \vec{F}_{1e} + \vec{F}_{2e} = \vec{F}_e \quad \cdots (7)$$

where $\vec{F}_e$ is net external force on system of two particles.

Let the total mass of the system be equal to.

$$M = m_1 + m_2 \quad \cdots (8)$$

Multiplying & dividing eqn (7) by M in L.H.S.

$$M\,\frac{(m_1\ddot{x}_1 + m_2\ddot{x}_2)}{M} = \vec{F}_e \quad \cdots (9)$$

$$M\ddot{X} = \vec{F}_e \quad \cdots (10)$$

$$M\left(\frac{m_1\frac{d^2x_1}{dt^2} + m_2\frac{d^2x_2}{dt^2}}{M}\right) = \vec{F}_e$$

$$M\frac{d^2}{dt^2}\left(\frac{m_1x_1 + m_2x_2}{M}\right) = \vec{F}_e \Rightarrow M\frac{d^2X}{dt^2} = \vec{F}_e$$

or. $M\ddot{X} = \vec{F}_e$

where $X = \dfrac{m_1x_1 + m_2x_2}{M} \Rightarrow \dfrac{m_1x_1 + m_2x_2}{m_1 + m_2} \quad \cdots (11)$

is called the center of mass.

$$X = \frac{m_1x_1}{m_1+m_2} + \frac{m_2x_2}{m_1+m_2} \quad \cdots (12)$$

$m_1 = m_2 = m$, then $M = 2m$ and.

$$X = \frac{x_1 + x_2}{2} \quad \cdots (13)$$

If $m_1 > m_2$, then X is closer to x_1 and vice-versa

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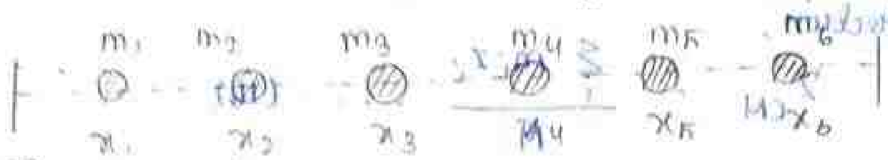
In general, the CM for a system of N number of particles is obtained by extending eqn (1) as.

$$X = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 + \dots + m_N x_N}{m_1 + m_2 + m_3 + \dots m_N}$$



$$X = \frac{\sum_{i=1}^{N} m_i x_i}{M} \quad (\because \sum m_i = M)$$

for example in a one-dimension, the CM is represented as.

$$X = \lim_{\Delta m \to 0} \frac{\sum_{i=1}^{N} \Delta m_i x_i}{M} \quad (14)$$

here, we consider an infinite sum. further for continuous distribution of mass, the summation is changed into integration

$$X = \frac{\int x \, dm}{\int dm} \quad (\because \int dm = M)$$

Example:-

So in the case of a uniform rod.

Mass per unit length of the rod $= \frac{M}{L}$.

Mass of the elemental length dx of the rod $dx = \left(\frac{M}{L}\right)dx$.

$$X = \frac{\int_0^L \frac{xM}{L} dx}{M} = \frac{1}{L}\left(\frac{x^2}{2}\right)\Big|_0^L = \frac{L}{2} \quad (15)$$

for N particles in three dimensions,

x direction y direction

$$x_{CM} = \frac{\sum_i m_i x_i}{M} \quad y_{CM} = \frac{\sum_i m_i y_i}{M}.$$

z direction,

$$z_{CM} = \frac{\sum_i m_i z_i}{M} \quad \ldots\ldots (16)$$

thus in vector notation $\vec{r_i} = x_i\hat{i} + y_i\hat{j} + z_i\hat{k}$.

the position of the CM is,

$$\vec{r}_{CM} = \frac{\sum_i m_i \vec{r_i}}{M}.$$

$$\vec{r}_{CM} = \frac{1}{M}\int \vec{r}\, dm \quad \ldots\ldots (17)$$

1.2 centre of Mass (CM)

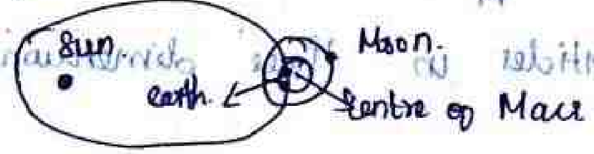
Definition :-

A point in the system at which whole mass of the body is supposed to be concentrated is called centre of mass of the body.

Examples for motion of centre of mass :-

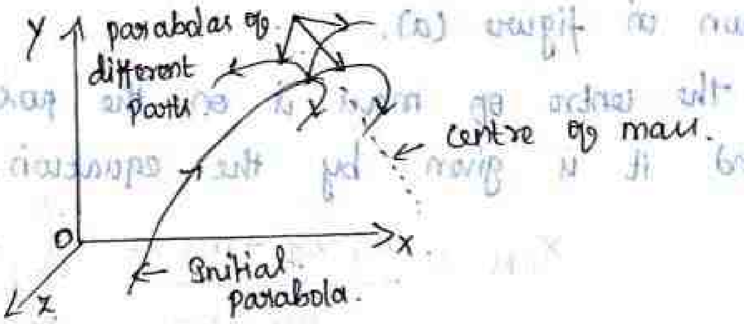
i) Motion of planets and the satellite

Let us consider the motion of the centre of mass of the earth and moon system. The moon moves round the earth in a circular orbit, the earth moves round the sun in the elliptical orbit.



ii) Projectile Trajectory:-

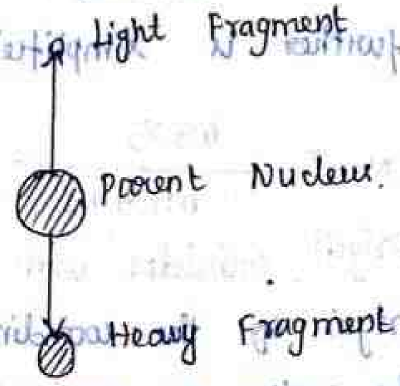
when a cracker is fired at an angle with the horizontal and when it explodes in the air, then different pieces of the cracker follows different parabolic paths.



iii) Decay of a Nucleus:-

Let us consider a spontaneous decay of a radio active nucleus into two fragments in figure. The two fragments move apart in opposite directions obeying the laws of conservations of energy and momentum.

The nucleus decays under the effect of internal forces. The heavier fragment moves with less speed while the lighter fragment moves with high speed.



Centre of mass of two point masses:-

with the equations for centre of mass, let us find the centre of mass of two point masses m_1 and m_2, which are at positions x_1 and x_2 respectively on the x-axis.

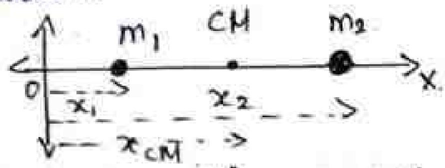
For this case, we can express the position of centre of mass in the following three ways based on the choice of the co-ordinate system.

i) when the masses are on positive x-axis :-

The origin is taken arbitrarily so that the masses m_1 and m_2 are at positions x_1 and x_2 on the positive x-axis as shown in figure (a).

The centre of mass is on the positive x-axis at x_{CM} and it is given by the equation.

$$x_{CM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

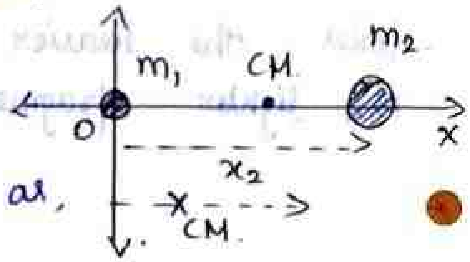


ii) when the origin coincides with any one of the masses :-

The calculation is minimised if the origin of the coordinate system is made to coincide with any one of the masses as shown in figure (b).

When the origin coincides with the point mass m_1, its position x_1 is zero (i.e., $x_1 = 0$). then,

$$x_{CM} = \frac{m_1(0) + m_2 x_2}{m_1 + m_2}$$



The equation further is simplified as,

$$x_{CM} = \frac{m_2 x_2}{m_1 + m_2}$$

iii) when the origin coincides with the centre of mass itself :-

If the origin of the coordinate system is made to coincide with the centre of mass then, $x_{CM} = 0$ and the mass m_1 is found to be on the negative x-axis as shown in figure (c). Hence, its position x_1 is negative $(-x_1)$.

$$0 = \frac{m_2(-x_1) + m_2 x_2}{m_1 + m_2}$$

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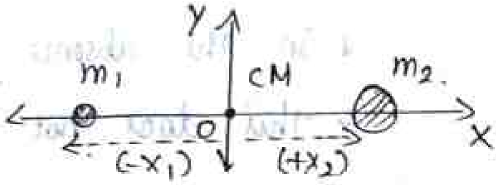
$$0 = m_1(-x_1) + m_2 x_2$$

$$m_1 x_1 = m_2 x_2$$



The equation given above is known as principle of moments.

1.3 Motion of centre of Mass :-

When a rigid body moves, it centre of mass will also move along the body.

For kinematic quantities like velocity ($\vec{V}_{CM}$) and acceleration ($\vec{a}_{CM}$) of the centre of mass with respect to time once and twice respectively.

For simplicity, let us take the motion along x direction only.

$$\vec{V}_{CM} = \frac{d\vec{x}_{CM}}{dt} = \frac{d}{dt}\left(\frac{\Sigma m_i x_i}{\Sigma m_i}\right)$$

differentiation with 'x'

$$= \frac{\Sigma m_i \left(\frac{d\vec{x}_i}{dt}\right)}{\Sigma m_i} \qquad \therefore \frac{dx}{dt} = \vec{v}$$

$$\boxed{\vec{V}_{CM} = \frac{\Sigma m_i \vec{V}_i}{\Sigma m_i}} \quad \ldots (1)$$

$$\vec{a}_{CM} = \frac{d}{dt}\left(\frac{d\vec{x}_{CM}}{dt}\right) = \left(\frac{d\vec{V}_{CM}}{dt}\right)$$

$$= \frac{\Sigma m_i \left(\frac{d\vec{v}_i}{dt}\right)}{\Sigma m_i} \qquad \therefore \frac{dv}{dt} = a$$

$$\boxed{\vec{a}_{CM} = \frac{\Sigma m_i \vec{a}_i}{\Sigma m_i}} \quad \ldots (2)$$

* In the absence of external force $\vec{F}_{ext} = 0$,
* This does not affect the position of the centre of mass.
* This mean that the centre of mass is in a state of rest or uniform motion.

Hence $\vec{a}_{CM}$ is zero, when centre of mass is at rest or has uniform motion ($\vec{v}_{CM} = 0$ (or) $\vec{v}_{CM}$ = constant).

There is no acceleration of centre of mass ($\vec{a}_{CM} = 0$)

From eqn (1) and (2).

$$\vec{v}_{CM} = \frac{\Sigma m_i \vec{v}_i}{\Sigma m_i} = 0 \quad (\text{or}) \quad \vec{v}_{CM} = \text{constant}.$$

It shows that,

$$\boxed{\vec{a}_{CM} = \frac{\Sigma m_i \vec{a}_i}{\Sigma m_i} = 0} \quad \ldots (3).$$

In the presence of external force, (ie $\vec{F}_{ext} \neq 0$), the centre of mass of the system will accelerate as given by the following equation.

$$\vec{F}_{ext} = (\Sigma m_i)\, \vec{a}_{CM}$$

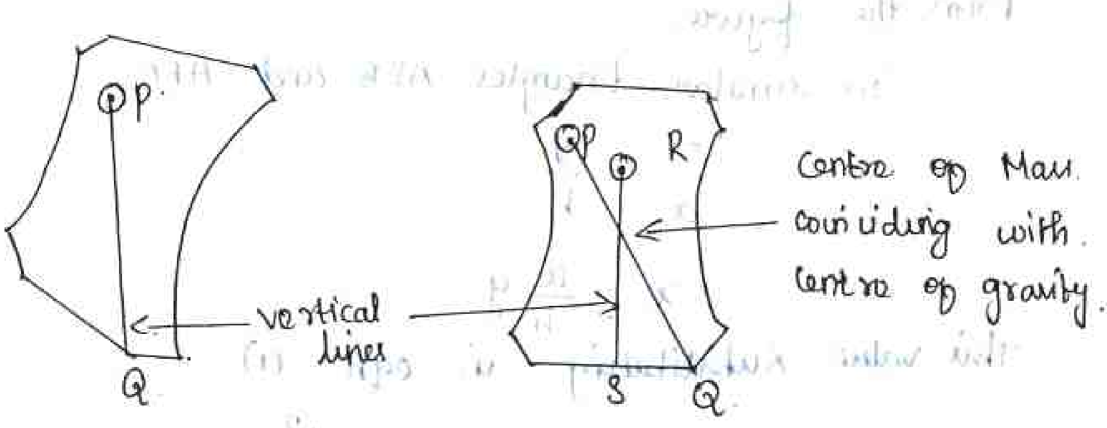
$$\vec{F}_{ext} = M\vec{a}_{CM}; \qquad \because \Sigma m_i = M$$

$$\boxed{\vec{a}_{CM} = \frac{\vec{F}_{ext}}{M}} \quad \ldots (4)$$

4). Centre of Mass (CM) of continuous Bodies (Rigid bodies)

Experimental location of the centre of mass:-

* The centre of masses of homogenous and regular shaped bodies coincides with their geometrical centre.
* Hence it can be easily located; but if the body is of irregular shape the location of its centre of mass is difficult.
* As an illustration, we describe below the method of suspension to locate the centre of mass of any regular or irregular shaped body.
* The body is first hung from some point P and a vertical line PQ is drawn.
* When the body is in equilibrium.
* The body is then hung from some other point R and a vertical line RS is drawn. (Fig)
* The point of intersection 'C' of these two lines PQ and RS gives the position of centre of mass.



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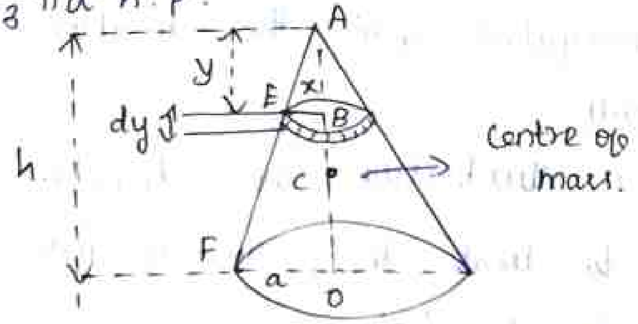
Centre of mass of a Solid cone :-

Fig. shows a right circular soid cone of base radius a and height h. Let ρ be the density of the material of the cone.

If the solid cone is homogeneous, then its mass $m = \frac{1}{3}\pi a^2 h . \rho$.



The centre of mass lies on the axis of symmetry AO.

The cone is considered to be made up of large number of circular discs, each of thickness dy.

Let us concider one such elementry disc of radius x at a distance y from the vertex A of the soid cone.

The mass of this elementary disc is,

$$dm = \rho\,(\pi x^2)\,dy \quad \cdots (1)$$

From the figure,

In similar triangles AEB and AFO.

$$\frac{x}{a} = \frac{y}{h}$$

$$x = \frac{a}{h}y.$$

This value substituting in eqn (1).

$$\therefore dm = \rho\,\pi\left(\frac{a}{h}y\right)^2 . dy \quad \cdots (2)$$

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Now from eqn (1), we have for the distance of centre of mass on the axis of symmetry AO as measured from the vertex A as,

$$Y_{CM} = \frac{1}{M}\int y\, dm \quad \ldots (3)$$

on substituting the value of eqn (2) in dm, we get,

$$Y_{CM} = \frac{1}{M}\int_0^h y\, \rho\pi \left(\frac{a}{h}y\right)^2 dy$$

(or)
$$Y_{CM} = \frac{\rho\pi a^2}{Mh^2}\int_0^h y^3\, dy \quad \ldots (4)$$

where the limit of y is taken from y = 0 to y = h to cover the entire solid cone filled with such elementary discs. It gives

$$Y_{CM} = \frac{\rho\pi a^2}{Mh^2}\left[\frac{y^4}{4}\right]_0^h$$

$$= \frac{\rho\pi a^2}{M\cancel{h^2}}\frac{\cancel{h^4}^{2}}{4} = \frac{\rho\pi a^2 h^2}{4M} \quad \ldots (5)$$

but M = total mass of the solid cone $= \frac{1}{3}\pi a^2 h\rho$

$$= \frac{\rho\pi a^2 h^2}{4M} = \frac{\cancel{\rho\pi a^2} h^{\cancel{2}}\cdot 3}{4\cdot\cancel{\pi a^2}\cancel{h\rho}}$$

the CM of cone from its vertex Y_{CM} is written as R_{CM}.

$$\boxed{R_{CM} = \frac{3}{4}h}$$

Thus, CM of a solid cone is at a distance of $\frac{3}{4}h$ from vertex of the cone along its axis.

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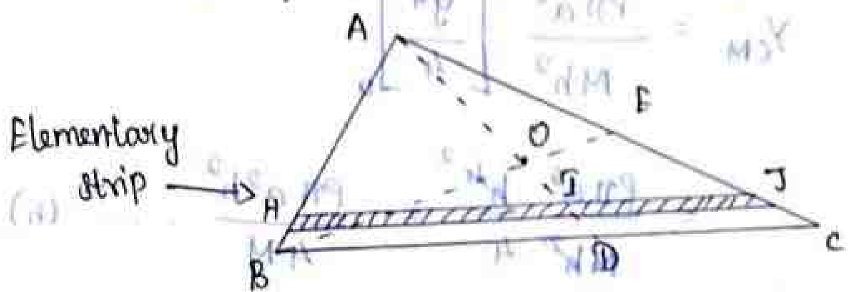
Centre of mass of a triangular lamina:-

* The medians of the triangle are axes of symmetry in the case of triangular sheets.
* We simply draw any two medians of the triangle which intersect at a point.
* This point is the centre of mass of the triangular body.
* We know that the medians bisect each other in the ratio of 2:1.
* The position of centre of mass on any median is at obtained by dividing that median in the ~~ratio~~ ratio 2:1
* The larger portion being towards the vertex.
* That point is the centre of mass.



AO : OD = 2:1

BO : OE = 2:1

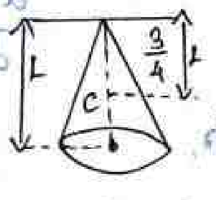
HI = IJ

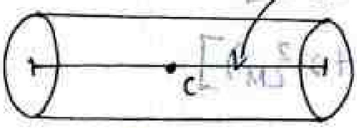
However, the position of centre of mass can also be calculated by assuming the triangle to be made up of large number of strips.

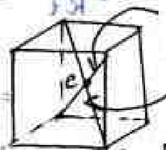
Parallel to one side of the triangle and placed one above the other as shown in fig.

Centre of mass of some regular objects :-

* fig shows the centre of masses of some regular shaped homogeneous rigid bodies.
* for a rigid body, the centre of mass is a point at a fixed position with respect to the body as a whole.
* Depending on the shape of the body and the way the mass is distributed in it, the centre of mass is a point that way or may not be within the body.
* If the shape is symmetrical and the mass distribution is uniform,
* we can usually find the location of the centre of mass quite easily.
* For a long thin rod of uniform cross section and density, the centre of mass is at the geometrical centre.
* For a thin circular plane ring, it is again at the geometrical centre of the circle.
* For a flat circular disc or rectangle, again the centre of mass is at the geometrical centre.







5. Kinetic energy of the system of particles:-

Let there be 'n' number of particles in a system of particles and these particles possess some motion. The motion of the i^{th} particle of this system depends on the external force $\vec{F_i}$ acting on it.

Let at any time the velocity of i^{th} particle be $\vec{v_i}$ then its kinetic energy would be,

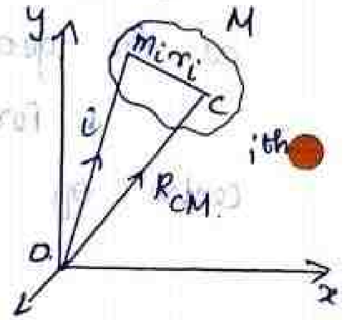
$$E_{ki} = \frac{1}{2} m v_i^2$$

$$E_{ki} = \frac{1}{2} m (v_i . v_i) \quad \text{(1)}.$$

Let $\vec{r_i}$ be the position vector of the i^{th} particle with respect to. O and $\vec{r_i}'$ be the position vector of the centre of mass w.r. to $\vec{r_i}$, as shown in figure, then,

$$\vec{r_i} = \vec{r_i}' + \vec{R_{CM}} \quad \ldots \text{(2)}.$$

where $\vec{R_{CM}}$ is the position vector of centre of mass of the system with respect to. O.



Differentiating the eqn (2) we get,

$$\frac{d\vec{r_i}}{dt} = \frac{d\vec{r_i}'}{dt} + \frac{d\vec{R_{CM}}}{dt}$$

or, $$v_i = v_i' + v_{CM} \quad \ldots \text{(3)}.$$

where v_i is the velocity of i^{th} particle and v_{CM} is the velocity of centre of mass of system of particle.

putting eqn (3) in (1) we get,

$$E_{Ki} = \frac{1}{2} m_i [(v_i' + v_{CM}) . (v_i' + v_{CM})]$$

$$= \frac{1}{2} m_i [(v_i'^2 + 2v_i' . v_{CM} + v^2_{CM})]$$

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$$E_{ki} = \frac{1}{2} m_i v_i'^2 + m_i v_i' \cdot v_{CM} + \frac{1}{2} m_i v_{CM}^2 \quad \ldots (4)$$

The sum of kinetic energy of all the particles can be obtained from eqn 4,

$$E_K = \sum_{i=1}^{n} E_{Ki} = \sum_{i=1}^{n} \left[\frac{1}{2} m_i v_i'^2 + m_i v_i' \cdot v_{CM} + \frac{1}{2} m_i v_{CM}^2\right]$$

$$E_K = \sum_{i=1}^{n} \frac{1}{2} m_i v_i'^2 + \sum_{i=1}^{n} m_i v_i' \cdot v_{CM} + \sum_{i=1}^{n} \frac{1}{2} m_i v_{CM}^2 \rightarrow (*)$$

rearrange ~~term~~ (*) eqn.

$$E_K = \frac{1}{2} v_{CN}^2 \sum_{i=1}^{n} m_i + \sum_{i=1}^{n} \frac{1}{2} m_i v_i'^2 + v_{CM} \sum_{i=1}^{n} m_i v_i'$$

$$E_K = \frac{1}{2} v_{CM}^2 M + \sum_{i=1}^{n} \frac{1}{2} m_i v_i'^2 + v_{CM} \frac{d}{dt} \sum_{i=1}^{n} m_i \vec{r}_i' \quad \ldots (5)$$

Now last term in eqn (5) is equal to zero. $\left(\because v_i' = \frac{dr_i'}{dt}\right)$

i.e., $\sum_{i=1}^{n} m_i \vec{r_i'} = 0$ $\qquad \left(\because \vec{r_i'} = r_i - R_{CM}\right)$

$$\therefore \sum_{i=1}^{n} m_i \vec{r_i'} = \sum_{i=1}^{n} m_i (\vec{r_i} - \vec{R}_{CM})$$

$$= \sum_{i=1}^{n} m_i \vec{r_i} - \sum_{i=1}^{n} m_i \vec{R}_{CM}$$

$$= MR_{CM} - MR_{CM} = 0 \qquad \left(\because \sum m_i \vec{r_i} = M\vec{R}_{CM}\right)$$

Therefore, kinetic energy of the system of particles is,

$$E_K = \frac{1}{2} M \vec{v}_{CM}^2 + \frac{1}{2} \sum_{i=1}^{n} m_i \vec{v_i'}^2 = E_{K_{CM}} + E_K' \quad \ldots (6)$$

where,

$$\boxed{E_{K_{CM}} = \frac{1}{2} \vec{v}_{CM}^2 M}$$

is the kinetic energy obtained as if all the mass were concentrated at the centre of mass.

$$\boxed{E_K' = \sum_{i=1}^{n} \frac{1}{2} m_i \vec{v_i'}^2} \quad \ldots (7)$$

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(17)

is the kinetic energy of the system of particle with respect to the centre of mass.

Hence it is clear from eqn (6) that kinetic energy of the system of particles consists of two parts.

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6. Rotation of Rigid Bodies:-

Rigid body:-

A rigid body is defined as the body which does not undergo any change in shape or volume when external forces are applied on it.

When forces are applied on a rigid body, the distance between any two particles of the body will remain unchanged however large the forces may be.

The solids, in which the changes produced by external forces are negligibly small, are usually considered as rigid body.

Rotational motion:-

* When a body rotates about a fixed axis, the motion is known as rotatory motion.

* A rigid body is said to have pure rotational motion if every particle of the body moves in a circle, the centre of which lies on a straight line called the axis of rotation in fig.

* The axis of rotation may lie inside the body or even outside the body

* The particles lying on the axis of rotation remains stationary.

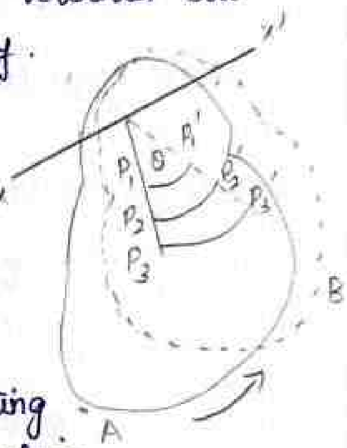
* Let this consider a rigid body that rotates about a fixed axis XOX' passing

through o and perpendicular to the plane of the paper as shown in figure.

Let the body rotate from the position 'A' to the position 'B'.

The different particles at $P_1, P_2, P_3, \ldots$ in the rigid body covers unequal distances $P_1P_1', P_2P_2', P_3P_3' \ldots$ in the same interval of time.

Thus, in the case of rotational motion, different constituent particles have different linear velocities but all of them have the same angular velocity.



Equation of rotational motion:-

As in linear motion, for a body having uniform angular acceleration we shall derive the equations of motion.

Let us consider a particle start rotating with angular velocity ω_0 and angular acceleration α. At any instant t,

Let ω be the angular velocity of the particle and θ be the angular displacement produced by the particle

therefore change in angular velocity in time $t = \omega - \omega_0$

but, angular acceleration = $\dfrac{\text{change in angular velocity}}{\text{time taken}}$

ie,

$$\alpha = \frac{\omega - \omega_0}{t} \quad \ldots (1)$$

$$\alpha t = \omega - \omega_0$$

(or) $$\boxed{\omega = \omega_0 + \alpha t} \quad \ldots (2)$$

Average angular velocity $= \left(\dfrac{\omega + \omega_0}{2}\right)$

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Total anglar displacement = average angular velocity x time taken

(i.e) $\theta = \left(\frac{\omega + \omega_0}{2}\right)t$ . . . (3).

Substituting ω from the eqn (2).

$$\theta = \left(\frac{\omega_0 + \alpha t + \omega_0}{2}\right)t \Rightarrow \left(\frac{2\omega_0 + \alpha t}{2}\right)t$$

$$= \left(\frac{2\omega_0}{2} + \frac{\alpha t}{2}\right)t$$

$$\boxed{\theta = \omega_0 t + \frac{1}{2}\alpha t^2} \quad \text{. . . . (4)}$$

From eqn (1), $t = \frac{\omega - \omega_0}{\alpha}$ . . . (5).

Using equation (5) in (3)

$$\theta = \left(\frac{\omega + \omega_0}{2}\right)\left(\frac{\omega - \omega_0}{\alpha}\right) = \left(\frac{\omega^2 - \omega_0^2}{2\alpha}\right)$$

$$2\alpha\theta = \omega^2 - \omega_0^2$$

$$\boxed{\omega^2 = \omega_0^2 + 2\alpha\theta} \quad \text{. . . . (6).}$$

The equations (2), (4) and (6) are the equations of rotational motion.

7. Rotational kinematics:-

Rotational kinetic Energy and Moment of inertia:-

* Consider a rigid body rotating about a fixed axis XOX'.

* The rigid body consists of a large number of particles. situated at distances. Let m_1, m_2, m_3 . . . etc.

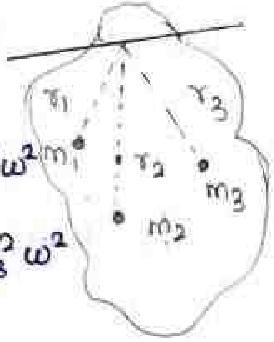
* Be the masses of the particles situated at distances. $r_1, r_2, r_3, \ldots$ etc. from the fixed axis.

kinetic Energy of first particle $= \frac{1}{2} m_1 v_1^2$

(13)

$= \frac{1}{2} m (r_1 \omega)^2$ $\therefore (v_1 = r_1 \omega)$

$\therefore$ Kinetic energy of the first particle $= \frac{1}{2} m_1 r_1^2 \omega^2$



Similarly.

kinetic energy of the Second particle $= \frac{1}{2} m_2 r_2^2 \omega^2$

kinetic energy of the third particle $= \frac{1}{2} m_3 r_3^2 \omega^2$

$\therefore$ kinetic energy of the rigid body.

$$E_K = \frac{1}{2} m_1 r_1^2 \omega^2 + \frac{1}{2} m_2 r_2^2 \omega^2 + \frac{1}{2} m_3 r_3^2 \omega^2 + \dots$$

$$= \frac{1}{2} \omega^2 (m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \dots)$$

$$E_K = \frac{1}{2} \omega^2 \Sigma m r^2$$

The term $\Sigma m r^2$ is called moment of inertia of a body about the given axis of rotation and denoted by I.

ie, $I = \Sigma m r^2$

$\therefore$ the kinetic energy of the rigid body $E_k = \frac{1}{2} \omega^2 I$

ie, $\boxed{E_k = \frac{1}{2} I \omega^2}$

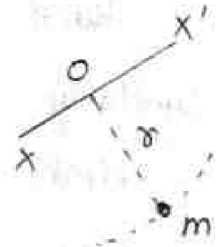
Moment of inertia or Rotational inertia:-

* There is a tendency to resist changes in uniform rotational motion. eg:- If a fan is switched off it continues to rotate for some more time before it be comes to rest.

* The property of a body by which it resists change uniform rotational motion is called rotational inertia or moment of inertia.

Moment of inertia of a particle:-

The moment of inertia of a particle about an axis is defined as the product of the mass of the particle and square of the distance of the particle from the axis of rotation.



'm' is the mass of the particle.

'r' is the distance of the particle

the moment of inertia of the particle.

$$\boxed{I = mr^2}$$

The unit of moment of inertia is $kg\,m^2$.

Moment of inertia of a rigid body:-

consider a rigid body of mass M, rotating about an axis xx'. The body is supposed to be made of a large number of particles.

$$I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \cdots$$

$$I = \Sigma m r^2 \quad \cdots \quad (1)$$

The unit of moment of inertia is $kg m^2$.

when angular velocity $\omega = 1$ radian / sec.

Rotational kinetic energy $= E_R = \frac{1}{2} I \times \omega^2$

$$= \frac{1}{2} I \times 1^2 = \frac{1}{2} I \quad \cdots (2)$$

$$I = 2E_R$$

It shows that moment of inertia of a body is equal to twice the kinetic energy of a rotating body when angular velocity is one radian per second.

~~Pr~~ Practical utility of Moment of Inertia:-

1. Fly-wheel:

A fly-wheel is such a heavy wheel whose most of the mass is concentrated at the rim, so that its moment of inertia is quite large.

2. Wheels of vehicles:-

In cycle, riksha, car, motor car, scooter, etc. the moment of inertia of wheels is increased by concentrating most of the mass at the rim of the wheel and connecting the rim to the axle of the wheel through the spokes.

Radius of Gyration:-

$$I = M.k^2 \quad \ldots (3).$$

k is known as radius of gyration.

Difinition:-

The radius of gyration is defined as the distance from the axis of rotation.

$$I = \Sigma mr^2$$

$$= mr_1^2 + m_2r_2^2 + m_3r_3^2 + m_4r_4^2 \ldots + mr_n^2.$$

Multiplying and dividing by n,

$$= nm\left[\frac{r_1^2 + r_2^2 + r_3^2 + \ldots + r_n^2}{n}\right] = Mk^2.$$

$$M = nm. \Rightarrow Mk^2 = \frac{MR^2}{2}.$$

$$k^2 = \frac{R^2}{2} \Rightarrow k = \frac{R}{\sqrt{2}}.$$

8. Theorems on Moment of inertia:- (MI).

There are two important theorems which help to find the moment of inertia of a body.

They are,

1. parallel axes theorem and
2. perpendicular axes theorem.

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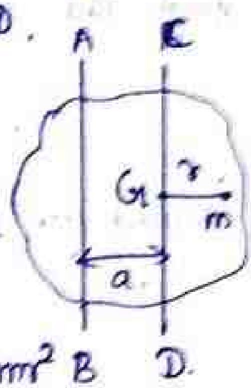
1. Theorem of parallel axes:-

Statement:-

The moment of inertia of a body about any axis is equal to the sum of its moment of inertia about a parallel axis passing through its centre of gravity of the body and the product of its mass of the body with the square of the distance between the two axes.

Explanation:-

Let G be the centre of gravity of a rigid body of Mass M. AB be an axis parallel to axis CD.



$$I = I_G + Ma^2.$$

proof:-

consider a particle of mass m at a distance r from CD.

M.I. of this particle about the axis CD $= mr^2$

$\therefore$ M.I. of the whole body about CD. $I_G = \Sigma mr^2$

M.I. of the particle about the axis AB $= m(r+a)^2$.

$\therefore$ MI of the whole body about AB, $I = \Sigma m(r+a)^2$.

$$I = \Sigma m(r^2 + a^2 + 2ar)$$

$$I = I_G + Ma^2 + 2a\Sigma mr$$

Since the body always balances above an axis through its centre of gravity. Σmr should be zero.

$$\boxed{I = I_G = Ma^2}$$

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2. Theorem (principle) of perpendicular axes:-

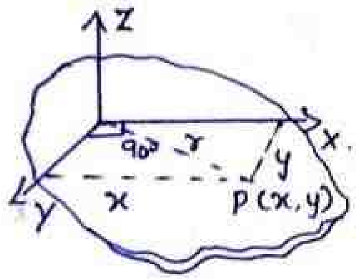
Statement:-

It states that the moment of inertia of a plane lamina about an axis perpendicular to its plane is equal to the sum of the moments of inertia of the plane lamina about any two mutually perpendicular axis in its own plane and intersecting each other at the point where the perpendicular axis passes through it.

Explanation:-

Let OX and OY be two mutually perpendicular axes in the plane of the lamina, intersecting each other at the point O. OZ is the axes perpendicular to both OX and OY.

$$\boxed{I_z = I_x + I_y}$$



proof:-

M.I. of this particle about OX = my^2.

M.I of the entire lamina about OX,

$$I_x = \Sigma my^2.$$

Similarly M.I. of the lamina about OY, $I_y = \Sigma mx^2$

M.I of the lamina about OZ axis through 'O' and perpendicular to the plane of the lamina $I_z = \Sigma mr^2$ ...(1)

$$r^2 = x^2 + y^2$$

Substituting eqn (2) in eqn (1),

$$I_z = \Sigma m(x^2+y^2)$$

$$I_z = \Sigma mx^2 + \Sigma my^2.$$

$$I_z = I_x + I_y.$$

$$\boxed{I_z = I_x + I_y}$$

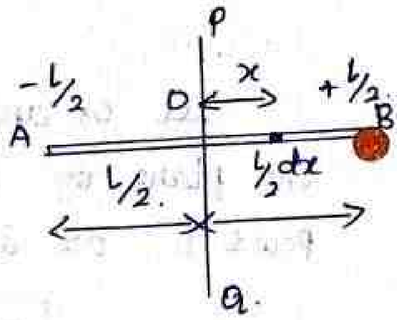
9. Moment of inertia of continuous bodies (Rigid Bodies)

1. Moment of inertia of a thin uniform rod :-

a) About an axis through its centre and perpendicular to its length.

Let AB be a thin uniform rod of length L and mass M. The rod is free to rotate about an axis PQ perpendicular to the length and passing through its centre 'O'.

Mass per unit length of the rod,



$$m = \frac{M}{L} \quad \ldots (1)$$

Mass of the element $= m.dx$

M.I of this element about the axis PQ

$= \text{mass} \times (\text{distance})^2$

$= mdx \cdot x^2$

$= mx^2 dx \quad \ldots (2)$

$x = -\frac{L}{2}$ and $x = \frac{L}{2}$

(ie) $$I = \int_{-L/2}^{+L/2} mx^2 dx \quad \ldots (3)$$

$$= m\left[\frac{x^3}{3}\right]_{-L/2}^{L/2} \Rightarrow m\left[\frac{(L/2)^3 - (-L/2)^3}{3}\right] = m\frac{\left[\frac{L^3}{8}+\frac{L^3}{8}\right]}{3}$$

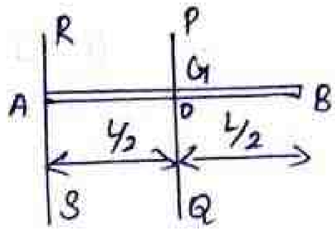
$$= \frac{m}{3}\left[\frac{2L^3}{8}\right] = \frac{\cancel{2}mL^3}{\cancel{24}_{12}} = \frac{mL^3}{12} \quad (\because mL = M)$$

$$I = mL \cdot \frac{L^2}{12}$$

$$\boxed{I = \frac{ML^2}{12}} \quad \ldots (4)$$

b) About an axis passing through one end of the rod and perpendicular to its length.

M.I. of the rod length PQ $= \frac{ML^2}{12}$



$$I = \frac{ML^2}{12} + M.\left(\frac{L}{2}\right)^2$$

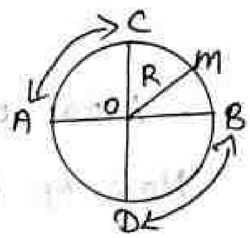
$$I = \frac{ML^2}{12} + \frac{ML^2}{4} = \frac{ML^2 + 3L^2}{12}$$

$$= \frac{\cancel{4}ML^2}{\cancel{12}\,3}$$

$$\boxed{I = \frac{ML^2}{3}}$$

2) Moment of inertia of thin ring (or hoop).

a) About an axis through its centre and perpendicular to its plane.



$$I = mR^2 \quad \ldots (1)$$

$\therefore$ M.I. of the ring about the axis,

$$I = \Sigma mR^2 \quad \ldots (2) \qquad \because \Sigma m = M.$$

$$\boxed{I = MR^2}$$

b) About a diameter :-

$$I_z = I_x + I_y \quad \ldots (3)$$

$$I_z = MR^2 \;\&\; I_x = I_y = I$$

$$\therefore MR^2 = I + I$$

(or)

$$2I = MR^2$$

$$\boxed{I = \frac{MR^2}{2}} \quad \ldots (4)$$

c) About a tangent in the plane of the ring :-

By parallel axis theorem, M.I about EF

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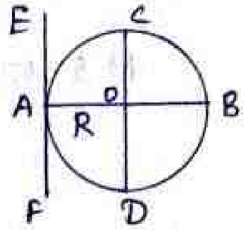
= MI about CD + Mass × Square of the distance OA



i.e. $I = \frac{MR^2}{2} + MR^2$

$= \frac{MR^2 + 2MR^2}{2}$

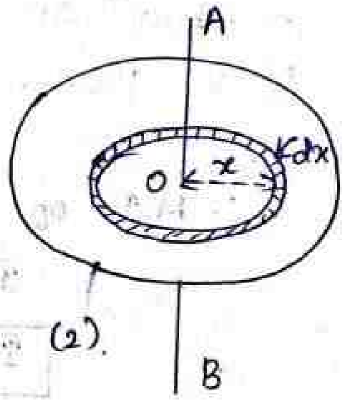
$I = \frac{3}{2} MR^2$ .... (5)

3. Moment of inertia of a thin circular disc:-

a) About an axis through its centre and perpendicular to its plane.

Mass per unit area of the disc $= \frac{M}{\text{Area of the disc}}$

$= \frac{M}{\pi R^2}$ .... (1)



Area of the strip $= 2\pi x dx$.

Mass of the strip $= \frac{M}{\cancel{\pi} R^2} 2\cancel{\pi} x dx$

$= \frac{2M}{R^2} x dx$ .... (2).

$= \frac{2M}{R^2} x dx \; x^2$.

$= \frac{2M}{R^2} x^3 dx$.

limits $x = 0$ & $x = R$

$I = \int_0^R \frac{2M}{R^2} x^3 dx$ ... (4)

$= \frac{2M}{R^2} \int_0^R x^3 dx \Rightarrow \frac{2M}{R^2} \left(\frac{x^4}{4}\right)_0^R$

$= \frac{2M}{\cancel{R^2}} \frac{R^{\cancel{4}\,2}}{\cancel{4}\,2} \Rightarrow \boxed{I = \frac{MR^2}{2}}$ ... (5).

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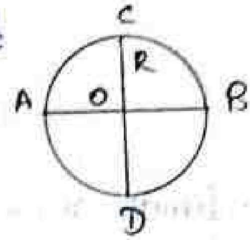
(31)

b) About a diameter:-



The moment of inertia of the disc about an axis, through the centre and perpendicular to the plane of the disc is.

$$= \frac{MR^2}{2} \quad \ldots (6)$$

$$I_z = I_x + I_y \quad \ldots (7)$$

$$I_z = \frac{MR^2}{2}$$

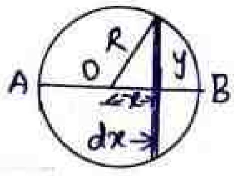
$$I_x = I_y = I \implies \frac{MR^2}{2} = I + I$$

$$\frac{MR^2}{2} = 2I$$

$$\boxed{I = \frac{MR^2}{4}} \quad \ldots (8)$$

4. Moment of inertia of solid sphere:

a) About a diameter:-



Mass per unit volume of the sphere.

$$\frac{Mass}{Volume} = \frac{M}{4/3 \pi R^3} = \frac{3M}{4\pi R^3}$$

the radius of this disc is given by $y^2 = (R^2 - x^2)$

Area of this disc $= \pi y^2 = \pi (R^2 - x^2)$

volume of the disc = Area x thickness $= \pi (R^2 - x^2) dx$ ... (2)

Mass of the elemental disc $= \frac{3M}{4\pi R} \times \pi (R^2 - x^2) dx$

$$= \frac{3M}{4R^3} (R^2 - x^2) dx \quad \ldots (3)$$

M. I of this disc about the axis AB.

$$= \frac{Mass \times (Radius)^2}{2}$$

$$= \frac{3M}{4R^3} (R^2 - x^2) dx \times \frac{y^2}{2}$$

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$$= \frac{3M}{4R^3}(R^2-x^2)\,dx \times \frac{R^2-x^2}{2} \qquad (\because y^2 = R^2-x^2)$$

$$= \frac{3M}{8R^3}(R^2-x^2)^2\,dx \quad \ldots\ldots (4)$$

limits $x = -R$ and $x = +R$

Moment of inertia of the solid sphere about a diameter.

$$I = \int_{-R}^{+R} \frac{3M}{8R^3}(R^2-x^2)^2\,dx \Rightarrow = 2\int_0^R \frac{3M}{8R^3}(R^2-x^2)^2\,dx$$

$$\cancel{2} \times \frac{3M}{\cancel{8}R^3}\int_0^R (R^2-x^2)^2\,dx$$

$$I = \frac{3M}{4R^3}\int_0^R (R^4 - 2R^2x^2 + x^4)\,dx \quad \ldots (5)$$

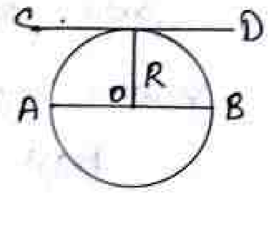
$$= \frac{3M}{4R^3}\left[R^4x - 2R^2\frac{x^3}{3} + \frac{x^5}{5}\right]_0^R$$

$$= \frac{3M}{4R^3}\left[R^5 - \frac{2R^5}{3} + \frac{R^5}{5}\right]$$

$$= \frac{3M}{4R^3}\left[\frac{15R^5 - 10R^5 + 3R^5}{15}\right] \Rightarrow = \frac{3M}{4R^3} \times \frac{8R^5}{15}$$

$$\boxed{I = \frac{2}{5}MR^2}$$

b) About a tangent :-



$$I = \frac{2}{5}MR^2 + MR^2$$

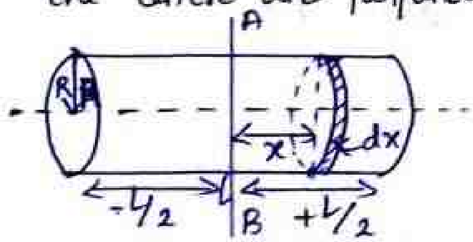
$$= \frac{2MR^2 + 5MR^2}{5}$$

$$\boxed{I = \frac{7}{5}MR^2}$$

5. Moment of Inertia of a solid cylinder :-

a) About an axis passing through the centre and perpendicular to its length.

$$m = \frac{M}{l} \quad \ldots (1)$$



(29)

Mass of the disc = mdx.

Moment of inertia of this disc about its own diameter,

$$= \frac{\text{Mass} \times (\text{Radius})^2}{4} = \frac{mdx\, R^2}{4}.$$

$$= \frac{mR^2 dx}{4} \quad \ldots (2).$$

Parallel diameter = Moment of inertia + Mass x Square of the distance.

$$= \frac{mR^2 dx}{4} + mdx\, x^2 \quad \ldots (3).$$

Limits $x = -l/2$ and $x = +l/2$

$\therefore$ M.I. of the cylinder about AB,

$$I = \int_{-l/2}^{+l/2} \left(\frac{mR^2 dx}{4} + mx^2 dx \right)$$

$$= \int_{-l/2}^{+l/2} \frac{mR^2 dx}{4} + \int_{-l/2}^{l/2} mx^2 dx.$$

$$= 2\int_0^{l/2} \frac{mR^2 dx}{4} + 2\int_0^{l/2} mx^2 dx$$

$$I = \frac{mR^2}{2} (x)_0^{l/2} + 2m\left(\frac{x^3}{3}\right)_0^{l/2} \quad \ldots (6).$$

$$= \frac{mR^2}{2} \times \frac{L}{2} + 2m \times \frac{l^3}{3\times 8}. \qquad \left(\because m = \frac{M}{L}\right)$$

$$= \frac{M}{L} \times \frac{R^2}{2} \times \frac{L}{2} + \frac{2M}{L} \times \frac{L^3}{3\times 8}$$

$$= \frac{MR^2}{4} + \frac{ML^2}{12} \Rightarrow \boxed{I = M\left(\frac{R^2}{4} + \frac{l^2}{12}\right)} \quad \ldots (7).$$

b) About the axis of the cylinder :-

$$= \frac{mR^2}{2} \quad \ldots (8).$$

$\therefore$ M.I. of the solid cylinder about its axis $= \Sigma \frac{mR^2}{2}$

$$\boxed{I = \frac{MR^2}{2}} \quad \ldots (9)$$

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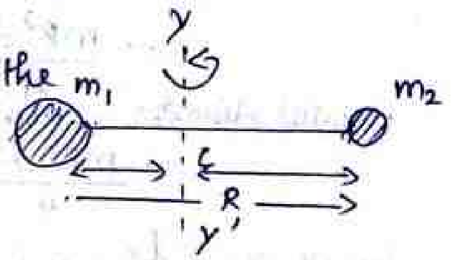
10. <u>Moment of inertia of a Diatomic Molecule :-</u>

A diatomic molecule, in its stable equilibrium position consists two atoms that are at a distance 'R' apart. The distance 'R' is called the bond length between the two atoms.

Let 'c' be the center of mass of the molecule and r_1 and r_2.



$$r_1 + r_2 = R \quad \ldots (1).$$

$$m_1 r_1 + m_2 r_2 \Rightarrow m_1 r_1 = m_2 r_2 \quad \ldots (2)$$

where m_1 and m_2 are the masses of two atoms respectively. From eqn (1).

$$r_1 = R - r_2 \quad \ldots (3).$$

and from eqn (2),

$$r_2 = \frac{m_1 r_1}{m_2} \quad \ldots (4)$$

$$r_1 = R - \frac{m_1 r_1}{m_2}$$

$$R = r_1 + \frac{m_1 r_1}{m_2} = r_1 \left[1 + \frac{m_1}{m_2}\right] \ldots (5)$$

(or)

$$r_1 = \frac{R}{\left[1 + \frac{m_1}{m_2}\right]} \quad \ldots (6).$$

The center of mass 'c' and perpendicular to the bond is given by,

$$I = m_1 r_1^2 + m_2 r_2^2 \quad \ldots (7).$$

$$I = m_1 r_1 . r_1 + m_2 r_1 . r_2.$$

(∵ from eqn (2))

$$I = m_1 r_1 (r_1 + r_2)$$

by using eqn (1). $I = m_1 r_1 R$

Substituting eqn (6) in eqn (8) gives,

$$I = m_1 R \left[\frac{R}{\left(1 + \frac{m_1}{m_2}\right)}\right]$$

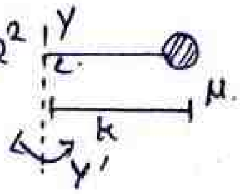
So,

$$I = \frac{m_1 R^2}{\left(1 + \frac{m_1}{m_2}\right)} = \frac{m_1 R^2}{\frac{m_2 + m_1}{m_2}}$$

$$= \frac{m_1 m_2 R^2}{m_2 + m_1}$$

(or)

$$I = \left(\frac{m_1 m_2}{m_1 + m_2}\right) R^2$$



$$\therefore \mu = \frac{m_1 m_2}{m_1 + m_2}$$

$$I = \mu R^2 \quad \ldots (9).$$

μ is called as reduced mass of the molecule.

Mass of the disc = mdx.

Moment of inertia of this disc about its own diameter,

$$= \frac{\text{Mass} \times (\text{Radius})^2}{4} = \frac{mdx\, R^2}{4}$$

$$= \frac{mR^2 dx}{4} \quad \ldots (2)$$

parallel diameter = Moment of inertia + Mass x square of the distance.

$$= \frac{mR^2 dx}{4} + mdx\, x^2 \quad \ldots (3)$$

Limits $x = -L/2$ and $x = +L/2$

∴ M.I. of the cylinder about AB,

$$I = \int_{-L/2}^{+L/2} \left(\frac{mR^2 dx}{4} + mx^2 dx \right)$$

$$= \int_{-L/2}^{+L/2} \frac{mR^2 dx}{4} + \int_{-L/2}^{L/2} mx^2 dx$$

$$= 2\int_0^{L/2} \frac{mR^2 dx}{4} + 2\int_0^{L/2} mx^2 dx$$

$$I = \frac{mR^2}{2}(x)_0^{L/2} + 2m\left(\frac{x^3}{3}\right)_0^{L/2} \quad \ldots (6)$$

$$= \frac{mR^2}{2} \times \frac{L}{2} + 2m \times \frac{L^3}{3 \times 8} \qquad \left(\because m = \frac{M}{L}\right)$$

$$= \frac{M}{L} \times \frac{R^2}{2} \times \frac{L}{2} + \frac{2M}{L} \times \frac{L^3}{3 \times 8}$$

$$= \frac{MR^2}{4} + \frac{ML^2}{12} \Rightarrow \boxed{I = M\left(\frac{R^2}{4} + \frac{L^2}{12}\right)} \quad \ldots (7)$$

b) About the axis of the cylinder :-

$$= \frac{mR^2}{2} \quad \ldots (8)$$

∴ M.I. of the solid cylinder about its axis $= \Sigma \frac{mR^2}{2}$

$$\boxed{I = \frac{MR^2}{2}} \quad \ldots (9)$$

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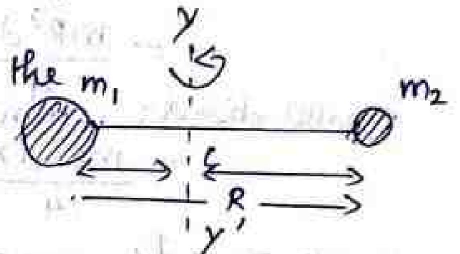
10. Moment of inertia of a Diatomic Molecule :-

A diatomic molecule, in its stable equilibrium position consists two atoms that are at a distance 'R' apart. The distance 'R' is called the bond length between the two atoms.

Let 'c' be the center of mass of the molecule and r_1 and r_2,



$$r_1 + r_2 = R \quad \ldots (1).$$

$$m_1 r_1 + m_2 r_2 \Rightarrow m_1 r_1 = m_2 r_2 \quad \cdots (2)$$

where m_1 and m_2 are the masses of two atoms respectively from eqn (1).

$$r_1 = R - r_2 \quad \ldots (3).$$

and from eqn (2),

$$r_2 = \frac{m_1 r_1}{m_2} \quad \ldots (4).$$

$$r_1 = R - \frac{m_1 r_1}{m_2}.$$

$$R = r_1 + \frac{m_1 r_1}{m_2} = r_1 \left[1 + \frac{m_1}{m_2}\right] \cdots (5)$$

(or)

$$r_1 = \frac{R}{\left[1 + \frac{m_1}{m_2}\right]} \quad \ldots (6).$$

The center of mass 'c' and perpendicular to the bond is given by,

$$I = m_1 r_1^2 + m_2 r_2^2 \quad \ldots (7).$$

$$I = m_1 r_1 \cdot r_1 + m_2 r_1 \cdot r_2.$$

(∵ from eqn (2))

$$I = m_1 r_1 (r_1 + r_2)$$

by using eqn (1). $I = m_1 r_1 R$

Substituting eqn (6) in eqn (8) gives,

$$I = m_1 R \left[\frac{R}{\left(1 + \frac{m_1}{m_2}\right)}\right]$$

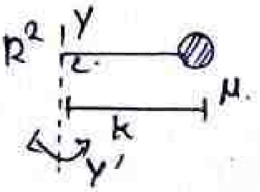
So,

$$I = \frac{m_1 R^2}{\left(1 + \frac{m_1}{m_2}\right)} = \frac{m_1 R^2}{\frac{m_2 + m_1}{m_2}}$$

$$= \frac{m_1 m_2 R^2}{m_2 + m_1}$$

(or)

$$I = \left(\frac{m_1 m_2}{m_1 + m_2}\right) R^2$$



$$\therefore \mu = \frac{m_1 m_2}{m_1 + m_2}$$

$$I = \mu R^2 \quad \ldots (9).$$

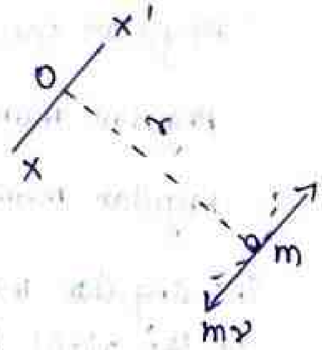
μ is called as reduced mass of the molecule.

$K = R$, which is called radius of gyration, so moment of inertia $I = \mu k^2$ ... (10)

11. Rotational Dynamics of Rigid Bodies :-

Angular momentum :-

The moment of linear momentum is known as angular momentum.



Angular momentum = Linear momentum x distance

$$\vec{L} = mv \times r \qquad (\because v = r\omega)$$

$$= mr\omega \times r$$

Angular momentum $= mr^2\omega$

where ω is the angular velocity of the particle.

$$\vec{L} = \vec{r} \times \vec{p}$$

S.I unit for angular momentum is $kg\,m^2 s^{-1}$

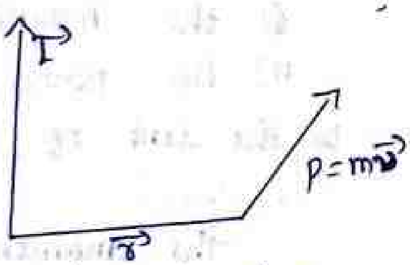
Definition :-

Angular momentum of a particle is defined as its moment of linear momentum.

It is given by the produced of linear momentum and perpendicular distance of its line of action from the axis of rotation. It is denoted by $\vec{L}$.

The vector product of $\vec{r}$ and $\vec{p}$, the linear momentum

$$\vec{L} = \vec{r} \times \vec{p} \quad \ldots (1)$$



$\vec{r} \rightarrow$ position vector

$\vec{p} \rightarrow$ linear momentum

The direction of angular momentum is perpendicular to plane containing $\vec{r}$ and $\vec{p}$.

Expression for angular momentum of a rigid body:-

Let $m_1, m_2, m_3, \ldots$ etc. be the masses of the particles situated at distances $r_1, r_2, r_3 \ldots$ etc. from the fixed axis.

Angular momentum = linear momentum x distance

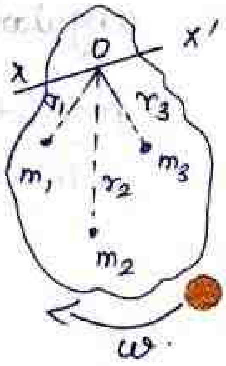
$$= mv \times r$$

$$= mr\omega \times r = mr^2\omega \quad (\because v = r\omega)$$

$\therefore$ Angular momentum of the first particle $= m_1 r_1^2 \omega$

Angular momentum of the second particle $= m_2 r_2^2 \omega$

Angular momentum of the third particle $= m_3 r_3^2 \omega$



$$\left.\begin{array}{l}\text{The angular momentum} \\ \text{of the rigid body}\end{array}\right\} = m_1 r_1^2 \omega + m_2 r_2^2 \omega + m_3 r_3^2 \omega$$

$$= \omega (m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2)$$

$$= \omega \Sigma m r^2$$

$$I = \Sigma m r^2$$

the angular momentum of the rigid body $= \omega I$

Angular momentum $L = I\omega$.

Torque (τ)

Torque is the turning effect of a force on a body on which the force acts.

The turning effect of a force depends on.

i) the magnitude of the force and,

ii) the perpendicular distance from the axis of rotation to the line of action of the force.

Definition:-

The moment of the applied force is called torque. It is represented by the symbol 'τ'.

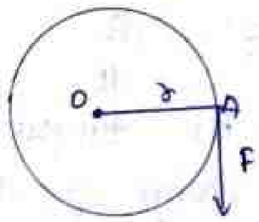
Torque = Force X distance

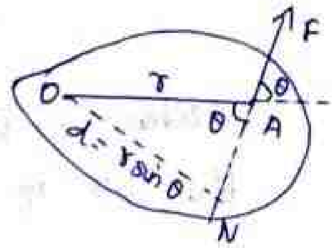
$$\tau = F \times r$$

Torque in vector notation:-

The ability of a force to rotate a body about an axis is measured in terms of a quantity called torque.

$\boxed{\text{Torque } \tau = \vec{r} \times \vec{F}}$





$\therefore$ torque $= F \times ON = F \times d$

$= F \cdot r \sin\theta$

This can be expressed in vector form as.

$$\vec{\tau} = \vec{r} \times \vec{F}$$

If $\vec{r}$ and $\vec{F}$ perpendicular to each other $\tau = rF$.

Unit of Torque:- Newton - metre (Nm)

Laws of motion applied to rotatory motion:-

* A rotating body tends to rotate continuously and uniformly about a fixed axis unless it is acted upon by an external torque.

* The rate of change of angular momentum is equal to the external torque applied and acts in the direction of torque.

* When a body exerts a torque on another body, the second body exerts an equal and opposite torque on the first body about the same axis of rotation.

Relation between torque and angular momentum:-

Torque = rate of change of angular momentum.

$$\tau = \frac{(I\omega - I\omega_0)}{t} = I\frac{(\omega - \omega_0)}{t} = I\alpha$$

where α is the angular acceleration.

$$\tau = I\alpha \qquad \left(\because \alpha = \frac{d\omega}{dt}\right)$$

$$\tau = I\frac{d\omega}{dt} \qquad (\because I\omega = L)$$

$$\tau = \frac{d}{dt}(I\omega) \Rightarrow \boxed{\tau = \frac{dL}{dt}}$$

18. Conservation of Angular momentum :-

The equation of motion of angular momentum of a particle is given by,

$$\vec{\tau} = \frac{d\vec{L}}{dt} \quad \ldots (1).$$

where $\vec{\tau}$ is the torque acting on the particle and $\frac{d\vec{L}}{dt}$ is the rate of change of angular momentum.

$$\vec{\tau} = 0$$

$$\frac{d\vec{L}}{dt} = 0 \quad (or) \quad \vec{L} = \text{constant}.$$

(i) angular momentum is conserved.

This is called law or principle of conservation of angular momentum.

Final angular momentum = Initial angular momentum.

(ii) Angular momentum remains unchanged.

Illustrations to explain the law of conservation of angular momentum.

i) consider a man holding a pair of dumb bells or weights in his outstretched arms extending and standing on a turn table capable rotating freely about a vertical axis passing through its centre.

As the man extends out his arms the M.I increases and the angular velocity of the turn table decreases to maintain the angular momentum a constant.

ii) In case of an acrobat in a circus, the acrobat leaves the swing with his arms and legs stretched. As soon as the leaves the swing, he possesses some angular momentum.

Hence the angular velocity of the acrobat increases considerably and the acrobat rolls.

13. Rotational Energy States of a Rigid Diatomic Molecule :-

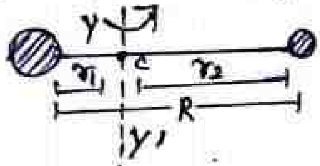
If the distance diatomic molecule rotates with respect to the centre of mass 'c', then its kinetic energy is given by,

$$E = \frac{1}{2} m_1 r_1^2 \omega^2 + \frac{1}{2} m_2 r_2^2 \omega^2 \quad \dots (1)$$

$$E = \frac{1}{2} (m_1 r_1^2 + m_2 r_2^2) \omega^2$$

$$E = \frac{1}{2} I \omega^2 \quad \dots (2) \qquad (\because I = m_1 r_1^2 + m_2 r_2^2)$$

The eqn (2) can be rewritten as.



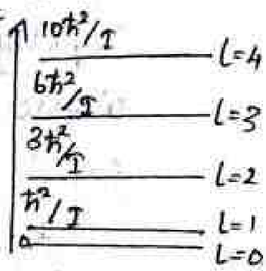
$$E = \frac{1}{2I} \cdot I^2 \omega^2 \quad \dots (3)$$

Then eqn (3) becomes. $(\because I\omega = L)$

$$E = \frac{L^2}{2I} \quad \dots (4)$$

$$L^2 = L(L+1)\hbar^2 \qquad L = 0, 1, 2 \dots \quad \dots (5)$$

where 'L' is the rotational quantum number.



$$E_L = \frac{L(L+1)\hbar^2}{2I} \quad \dots (6)$$

here $\hbar = \frac{h}{2\pi}$ and 'h' is the planck's constant.

Note that the levels are not equally spaced.

14. Gyroscope :-

Definition :-

A gyroscope is a device used for measuring or maintaining orientation and angular velocity.

It is spinning wheel or disc in which the axis of rotation is free to assume any orientation by itself.

Gyroscopic principle :-

All spinning objects have gyroscopic properties. The main properties that an object can experience in any

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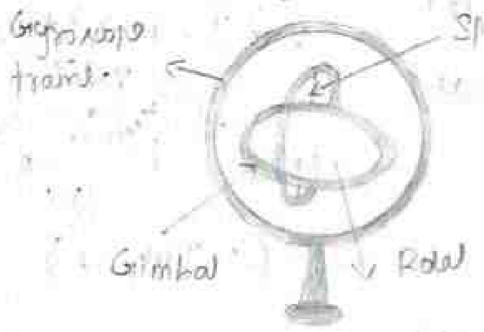
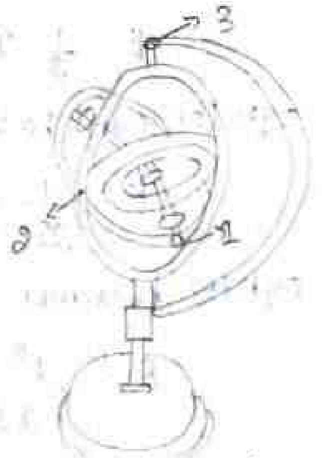
gyroscopic motion are rigidity in space and precession.

Description and working :-

A gyroscope is essentially a heavy wheel rotating at high speed about an axes passing through its centre of mass and so mounted as to be free to turn about any of the three mutually perpendicular axes 1, 2, 3





In other words, as long as the wheel rotates rapidly, it maintains its axis of rotation unchanged in space as the support is tilted in any manner.

Applications :-

* In view of the property of stability, the gyroscope used as stabilizers in ships, boats and aeroplanes.
* Due to the inherent stability of the gyroscope, it is used as a compass, and a gyro-compass is preferable to the magnetic compass in many respects.
* Another important application of the directional stability of a rapidly spinning body is the rifling of the barrels of the rifles.
* The rolling of hoops and the riding of bicycles are statically unstable since both of them cannot remain in equilibrium when at rest.

* This effect produces a movement of the plane of rotation, tending to counterbalance the disturbing action of gravity.

* Many modern aircraft instruments such as automatic pilot, bomb sights, artificial horizon, turn and bank indicators, etc. have been developed on gyroscope controlled principles.

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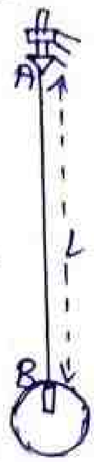
15. Torsion pendulum:-

* A circular metallic disc suspended using a thin wire that execute torsional oscillation is called torsional pendulum.

* It executes torsional oscillations, whereas a simple pendulum executes linear oscillations.

Description:-

A torsional pendulum consists of a metal wire suspended vertically with the upper end fixed.



Expression for the period of oscillation of a Torsion pendulum:-

The restoring couple set up in the wire $= C\theta$ .. (1)

where C - couple per unit twist.

If the disc is released, it oscillates with angular velocity $\frac{d\theta}{dt}$ in the horizontal plane about the axis of the wire.

These oscillations are known as torsional oscillations.

At Applied couple $= I\frac{d^2\theta}{dt^2}$ ... (2)

At equilibrium, applied couple = restoring couple.

$$I\frac{d^2\theta}{dt^2} = -C\theta$$

$$\frac{d^2\theta}{dt^2} = -\frac{C}{I}\theta \quad \text{.... (3)}$$

Hence, the motion of the disc being simple harmonic motion, the time period of the oscillation is given by.

$$T = 2\pi\sqrt{\frac{\text{displacement}}{\text{Acceleration}}}$$

$$= 2\pi\sqrt{\frac{\theta}{\frac{c}{I}\times\theta}}$$

$$\boxed{T = 2\pi\sqrt{\frac{I}{c}}} \quad \ldots (4)$$

Uses of Torsional pendulum:-

Torsional pendulum is used to determine,

* Rigidity modulus of the wire
* Moment of inertia of the disc
* Moment of inertia of an irregular body.

Determination of Rigidity Modulus of the wire:-

The rigidity modulus of the wire is,

$$T = 2\pi\sqrt{\frac{I}{c}} \quad \ldots (1)$$

Experiment:-

* A circular disc is suspended by a thin wire, whose rigidity modulus is to be determined.
* The length l of the wire is measured. This length is then changed by about 10 cm and the experiment is repeated.
* The disc is removed and its mass and diameter are measured

Time period of oscillation is,

$$T = 2\pi\sqrt{\frac{I}{c}} \quad \ldots (2)$$

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Squaring on both sides, we have

$$T^2 = 2^2\pi^2 \left(\sqrt{\frac{I}{c}}\right)^2 \quad \ldots (3)$$

$$T^2 = \frac{4\pi^2 I}{c} \quad \ldots (4)$$

Substituting couple per unit twist $c = \frac{\pi n r^4}{2L}$ in eqn ..(4)

$$T^2 = \frac{4\pi^2 I}{\frac{\pi n r^4}{2L}} = \frac{2L \times 4\pi^2 L}{\pi n r^4} \quad \ldots (5)$$

The rigidity modulus of the material of the wire,

$$\boxed{n = \frac{8\pi I}{r^4}\left(\frac{L}{T^2}\right)} \quad \ldots (6)$$

I - moment of inertia of circular disc $= \frac{MR^2}{2}$

where M - Mass of the circular disc

R - Radius of the disc.

16. Double pendulum:-

A double pendulum is a pendulum with another pendulum attached to its end.

It is a simple physical system that exhibits rich dynamic behaviour with strong sensitivity to initial conditions.

Kinematics of the double pendulum:-

* x = horizontal position of pendulum mass.
* y = vertical position of pendulum mass.
* θ = angle of pendulum.
* l = Length of rod.

$$x_1 = L_1 \sin\theta$$
$$y_1 = -L_1 \cos\theta_1$$

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$$x_2 = x_1 + L_2 \sin\theta_2$$
$$y_2 = y_1 - L_2 \cos\theta_2$$

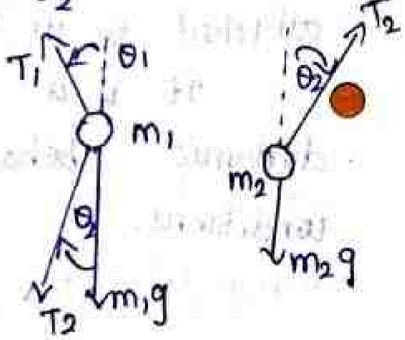
The velocity is the derivative with respect to time of the position,

$$\frac{dx_1}{dt} = \frac{d\theta_1}{dt} L_1 \cos\theta_1$$

$$\dot{x}_1 = \dot{\theta}_1 L_1 \cos\theta_1$$
$$\dot{y}_1 = \dot{\theta}_1 L_1 \sin\theta_1$$
$$\dot{x}_2 = \dot{x}_1 + \dot{\theta}_2 L_2 \cos\theta_2$$
$$\dot{y}_2 = \dot{y}_1 + \dot{\theta}_2 L_2 \sin\theta_2$$

The acceleration is the second derivative.

$$\ddot{x}_1 = -\dot{\theta}_1^2 L_1 \sin\theta_1 + \ddot{\theta}_1 L_1 \cos\theta_1$$
$$\ddot{y}_1 = \dot{\theta}_1^2 L_1 \cos\theta_1 + \dot{\theta}_1 L_1 \sin\theta_1$$
$$\ddot{x}_2 = \ddot{x}_1 - \dot{\theta}_2^2 L_2 \sin\theta_2 + \ddot{\theta}_2 L_2 \cos\theta_2$$
$$\ddot{y}_2 = \ddot{y}_1 \dot{\theta}_2^2 L_2 \cos\theta_2 + \dot{\theta}_2 L_2 \sin\theta_2$$



Forces in double pendulum :-

T = tension in the rod

m = mass of pendulum

g = acceleration due to gravity.

Here we show the net force and we Newton's law $F = ma$.

$$m_1 \ddot{x}_1 = -T_1 \sin\theta_2 + T_2 \sin\theta_2 \quad \text{--- (5)}$$
$$m_1 \ddot{y}_1 = T_1 \cos\theta_1 - T_2 \cos\theta_2 - m_1 g \quad \text{--- (6)}$$
$$m_2 \ddot{x}_2 = -T_2 \sin\theta_2 \quad \text{--- (7)}$$
$$m_2 \ddot{y}_2 = T_2 \cos\theta - m_2 g \quad \text{--- (8)}$$

uses of double pendulum :-

* The double pendulum is widely used in education research and applications.

* For example, the double pendulum is stable bench top experiment for introducing and studying chaos and state transitions.

* It has also been used to study chaos both experimently and numerically.

17. INTRODUCTION TO NON - LINEAR OSCILLATION :-

* Non - linear systems can show behaviours that linear systems cannot. These include ;

* Multiple steady state solutions, some stable and some unstable, in response to the same inputs.

* jump phenomena, involving discontinuous and significant changes in the response of the system as some forcing parameter is slowly varied.

* response at frequencies other than the forcing frequency.

* complex irregular motions that are extremely sensitive to initial conditions.

Non - Linear oscillators :-

A linear oscillator can oscillate with only one frequency, its motion is sinusoidal and periodic.

The position as a function of time will not be given by $y = A\cos(2\pi f_1 t)$, where $f_1 = \frac{1}{P}$, here P is period.

we can write symbolically as $P_n = \frac{P}{n}$ where n is an integer $n = 1, 2, 3 \ldots$ since the frequency f_n is the inverse of the period.

$$f_n = \frac{n}{p} = n\frac{1}{p} = nf_1 \quad \cdots (1).$$

The frequency f_1 is called the fundamental of the harmonic series.

$$y_n = A\cos\{n(2\pi f_1 t)\},\ n = 1, 2, 3, 4 \cdots (2).$$

If the period of the oscillations is P, then the frequencies present in the motion are,

$$f_1 = \frac{1}{P},\ f_2 = 2f_1 = \frac{2}{P},\ f_3 = 3f_1,\ f_4 = 4f_1 4f,\ \text{etc} \quad \cdots (3)$$

this is very different from the simple oscillator.

there are two important characterites of the nonlinear oscillators.

* The effects of the nonlinearity becomes much more important as the amplitude is increased.
* For some types of nonlinearity, the frequency of the oscillator will change with amplitude.

Applications and uses:-

Non-Linear system is a system in which the change of the output is not proportional to change of the input.

Non linear problems are of interest to engineers, biologists, physicists, mathematicians and many other scientists because most systems are inherently non-linear in nature.

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Unit - II

Electromagnetic waves

1. Maxwell's equations :-

* Faraday laid the foundation of electromagnetism.

* He showed that electromagnetic waves can be produced by changing electric and magnetic fields.

* Maxwell's equations form the foundation of electromagnetic theory.

Maxwell's Equations :- (Derivation)

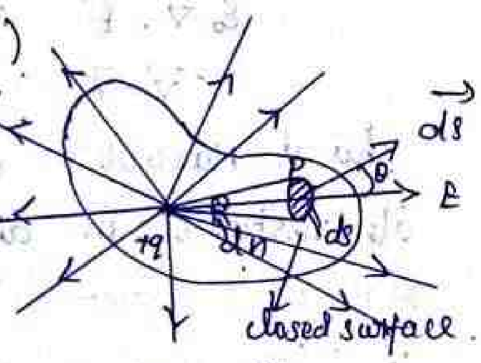
The first two equations are known as steady state equations and the last two equations are known as time varying equations.

Maxwell's Equation - I

(From Gauss's Law in electrostatics)

Integral form :-

According to Gauss law,



$$\oiint_S \vec{E}\cdot\vec{ds} = \frac{q}{\epsilon} \quad \ldots (1)$$

$(\because \vec{D} = \epsilon\vec{E})$

$$\oiint_S \epsilon\vec{E}\cdot\vec{ds} = q$$

$$\boxed{\oiint_S \vec{D}\cdot\vec{ds} = q \quad \ldots (2)}$$

ρ is the charge density.

Total energy inside the closed surface is given by,

$$q = \iiint_V \rho\, dv \quad \ldots (3)$$

Substituting eqn (3) in eqn (2)

$$\boxed{\oiint_S \vec{D}\cdot\vec{ds} = \iiint_V \rho\, dv} \quad \ldots (4)$$

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this is Maxwell's equation in integral form from Gauss law in electrostatics.

Applying Gauss's divergence theorem to LHS of eqn (4) we get,

i.e., $$\oint\!\!\oint_S \vec{D}\cdot\vec{ds} = \iiint_V \vec{\nabla}\cdot\vec{D}\,dv \quad \ldots (5)$$

on substituting eqn (5) in eqn (4) we get

$$\iiint_V \vec{\nabla}\cdot\vec{D}\,dv = \iiint_V \rho\,dv \quad \ldots (6)$$

(or) $$\boxed{\vec{\nabla}\cdot\vec{D} = \rho}$$

$$(\because D = \varepsilon_0\vec{E})$$

$$\vec{\nabla}\cdot\varepsilon_0\vec{E} = \rho$$

$$\varepsilon_0\vec{\nabla}\cdot\vec{E} = \rho$$

$$\vec{\nabla}\cdot\vec{E} = \rho/\varepsilon_0$$

This is Maxwell's equation from Gauss's law in electrostatics in differential form.

Maxwell's Equation - II
(from Gauss's law in magnetostatics)

Integral form:-

$$\boxed{\oint\!\!\oint_S \vec{B}\cdot\vec{ds} = 0} \quad \ldots (1)$$

this is Maxwell's equation in integral form from Gauss's Law in magnetostatics

Applying Gauss divergence theorem to the L.H.S of equation (1), we get,

$$\oint\!\!\oint_S \vec{B}\cdot\vec{ds} = \iiint_V \vec{\nabla}\cdot\vec{B}\,dv \quad \ldots (2)$$

on substituting eqn (2) in eqn (1), we have, ③

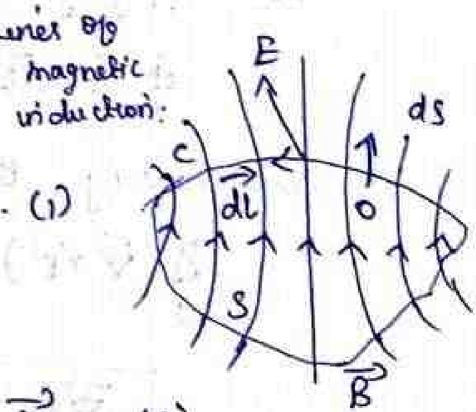
$$\iiint_V \vec{\nabla}\cdot\vec{B}\, dv = 0 \quad \ldots (3)$$

or $$\boxed{\vec{\nabla}\cdot\vec{B} = 0} \quad \ldots (4)$$

This is Maxwell's equations in differential form from Gauss's law in magnetostatics.

Maxwell's Equation - II

(From Faraday's Law)



Magnetic flux through a small area $ds = \vec{B}\cdot\vec{ds}$ ... (1)

$\therefore$ Total magnetic flux linked with the circuit $\Big\} = \phi_B$

$$= \oint_S \vec{B}\cdot\vec{ds} \ldots (2)$$

Faraday's Law states that the induced emf e is the rate of change of magnetic flux ϕ_B

$$\therefore e = -\frac{d\phi_B}{dt} = \frac{-d}{dt}\left[\oint_S \vec{B}\cdot\vec{ds}\right] \quad \ldots (3)$$

$$= \oint \frac{\partial\vec{B}}{\partial t}\cdot\vec{ds}$$

If $\vec{E}$ be the electric field strength, then, we know that $\vec{E} = \frac{dv}{dt}$

$$dv = \vec{E}\cdot\vec{dl}$$

$$\therefore V = \int dv = \int \vec{E}\cdot dl.$$

$$e = \oint \vec{E}\cdot\vec{dl} \quad \ldots (4)$$

Here, the integral is taken over a closed curve c.

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④

Equating eqn (3) and eqn (4), we have.

$$\boxed{\oint_C \vec{E}\cdot\vec{dl} = -\iint_S \frac{\partial \vec{B}}{\partial t}\cdot \vec{ds}} \quad \dots (5).$$

This is Maxwell's equation in integral form from Faraday's Law of electromagnetic induction.

Now applying Stoke's theorem to L.H.S of eqn (5).

$$\oint_C \vec{E}\cdot\vec{dl} = \iint_S (\vec{\nabla}\times\vec{E}).\vec{ds} \quad \dots (6).$$

on substituting the eqn (6) in eqn (5) we get.

$$\iint_S (\vec{\nabla}\times\vec{E})\,\vec{ds} = -\iint_S \frac{\partial \vec{B}}{\partial t}.\vec{ds} \quad \dots (7).$$

$$\boxed{\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}} \quad \dots (8).$$

Eqn (8) represents Maxwell's equation from Faraday's Law of electromagnetic induction in differential form.

<u>conduction current density</u>:

This is the current due to flow of electrons through the resistance in the circuit obeying ohm's law.

$$V = I_C R \quad \text{or} \quad I_C = \frac{V}{R} \quad \dots (1).$$

$$R = \frac{\rho L}{A} \Rightarrow R = \frac{L}{\sigma A} \quad \dots (2)$$

where, ρ - resistivity of the conductor

L - length of the conductor.

A - area of cross-section of conductor.

σ = conductivity of conductor $(\sigma = \frac{1}{\rho})$

Substituting for R from eqn (2) in eqn (1),

$$I_c = \frac{V}{\frac{L}{\sigma A}}$$

$$I_c = \frac{V\sigma A}{L} \quad \ldots (3)$$

$$\frac{I_c}{A} = \frac{V\sigma}{L} \Rightarrow \boxed{\vec{J_c} = \sigma\vec{E}} \quad \ldots (4)$$

$$\left(\because J_c = \frac{I_c}{A} \text{ and } E = \frac{V}{L}\right)$$

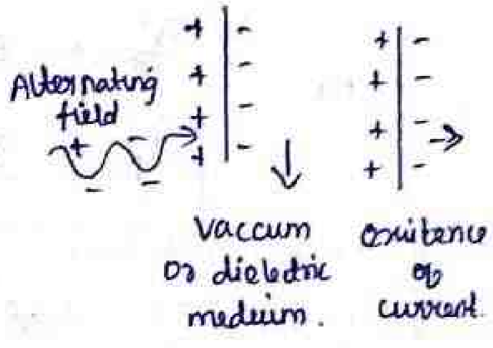
here, J_c represents the conduction current density.

Displacement current density :-

It results in the existense of current on the surface of capacitor. This current is called displacement current in the capacitor.

$$I_D = \frac{dq}{dt}$$

$$q = CV$$



C - capacitance of capacitor.

V - potential difference

$$\therefore I_D = \frac{d}{dt}(CV) = C\frac{dv}{dt} \quad \ldots (1)$$

The capacitance of parallel plate capacitor is given by.

$$C = \frac{\varepsilon A}{d} \quad \ldots (2).$$

where,

ε - permittivity of the medium.

A - Area of the parallel plate capacitor.

d - Distance between two plates.

Substituting for c from eqn (2) in eqn (1),

$$I_D = \frac{\varepsilon A}{d}\frac{dV}{dt} \quad \dots (3)$$

$$V = Ed \qquad \left(\because E = \frac{V}{d}\right) \quad \dots (4)$$

Substituting eqn (4) in eqn (3),

$$I_D = \frac{\varepsilon A}{d}\frac{d}{dt}(\vec{E}d)$$

$$I_D = \frac{\varepsilon A}{d} d \frac{d\vec{E}}{dt}$$

$$I_D = \varepsilon A \frac{d\vec{E}}{dt} \quad \dots (5)$$

$$\frac{I_D}{A} = \varepsilon \frac{d\vec{E}}{dt}$$

$$\vec{J}_D = \varepsilon \frac{d\vec{E}}{dt} \qquad \left(\because \frac{I_D}{A} = J_D\right)$$

where J_D displacement current density.

$$\vec{J}_D = \frac{d\varepsilon \vec{E}}{dt} \quad \dots (6)$$

or

$$\boxed{J_D = \frac{d\vec{D}}{dt}} \quad \dots (7) \qquad (\because D = \varepsilon E)$$

Maxwell's equation - IV

(from Ampere's circuital law)

$$\oint \vec{H}\cdot\vec{dl} = I \quad \dots (1)$$

current density $J = \frac{I}{A}$

where A is cross sectional area.

(or) $I = JA$ $\qquad \left(\because A = \iint_S ds\right)$

$$I = J\iint_S ds$$

(or) $$I = \iint_S \vec{J}\cdot\vec{ds} \quad \dots (2)$$

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Substituting eqn (2) in eqn (1), ⑦

$$\oint \vec{H} \cdot \vec{dl} = \iint_S \vec{J} \cdot \vec{ds} \quad \ldots (3)$$

Ampere's law is modified by introducing displacement current density.

$$\oint \vec{H} \cdot \vec{dl} = \iint_S (\vec{J_c} + \vec{J_D}) \vec{ds} \quad \ldots (4)$$

Substituting for $\vec{J_c} = \sigma \vec{E}$ and $\vec{J_D} = \frac{\partial \vec{D}}{\partial t}$

$$\oint \vec{H} \cdot \vec{dl} = \iint_S \left(\sigma \vec{E} + \frac{\partial \vec{D}}{\partial t} \right) ds$$

$$\oint \vec{H} \cdot \vec{dl} = \iint_S \left(\sigma \vec{E} + \varepsilon \frac{\partial \vec{E}}{\partial t} \right) ds \quad \ldots (5)$$

$$\oint \vec{H} \cdot \vec{dl} = \iint_S \left(\vec{J} + \frac{\partial \vec{D}}{\partial t} \right) ds \quad \ldots (6) \qquad (\because J = J_c)$$

$$(\because J = \sigma \vec{E}, \; D = \varepsilon \vec{E})$$

This is Maxwell's equation in integral form from Ampere's circuital law.

Applying Stoke's Theorem to L.H.S of eqn (6).

$$\oint \vec{H} \cdot \vec{dl} = \iint_S (\vec{\nabla} \times \vec{H}) \, ds \quad \ldots (7)$$

On substituting eqn (7) in eqn (6)

$$\iint_S (\vec{\nabla} \times \vec{H}) \, ds = \iint_S \left(\vec{J} + \frac{\partial \vec{D}}{\partial t} \right) ds \quad \ldots (8)$$

or

$$\boxed{\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}} \quad \ldots (9)$$

$$\boxed{\vec{\nabla} \times \vec{H} = \sigma \vec{E} + \varepsilon \frac{\partial \vec{E}}{\partial t}} \quad \ldots (10)$$

Equations (9) and (10) are Maxwell equations in differential form from Ampere's circuital Law.

Maxwell's equations in Free space:- ⑧

Four Maxwell's equations in differential from are.

$$\vec{\nabla}.\vec{D} = \rho \quad \ldots (1)$$

$$\vec{\nabla}.\vec{B} = 0 \quad \ldots (2)$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \ldots (3)$$

$$\vec{\nabla}\times\vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \quad \ldots (4)$$

Maxwell's equations reduce to.

$$\vec{\nabla}.\vec{D} = 0 \quad \ldots (5) \quad (\because \rho = 0)$$

$$\vec{\nabla}.\vec{B} = 0 \quad \ldots (6)$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \ldots (7)$$

$$\vec{\nabla}\times\vec{H} = \frac{\partial \vec{D}}{\partial t} \quad \ldots (8) \quad (\because J = 0)$$

Maxwell's equations in conducting Media:-

In conducting media.

$\vec{J} = \sigma\vec{E}$, where σ is electrical conductivity of the conducting medium.

$\vec{B} = \mu\vec{H}$, where μ is permeability of the medium

$\vec{D} = \varepsilon\vec{E}$, where ε is the permittivity of the conducting medium.

General Maxwells eqn reduce to,

$$\vec{\nabla}.\vec{D} = \rho \quad \ldots (1)$$

$$\vec{\nabla}.\vec{B} = 0 \quad \ldots (2)$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \ldots (3)$$

$$\nabla\times\vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \quad \ldots (4)$$

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⑨

Characteristics of Maxwell's equation :-

1. Maxwell's first Equation ; $\vec{\nabla}.\vec{E} = \frac{\rho}{\varepsilon_0}$.

* It explains Gauss's Law in electrostatics.
* It is time independent or steady state equation.
* charge acts as a source or sink for the lines of electric force.

2. Maxwell's Second Equation $\vec{\nabla}.\vec{B} = 0$.

* It expresses a well known observation that isolated magnetic poles do not exist.
* It is a time independent equation.
* It explains Gauss law in Magnetostatics.

3. Maxwell's Third Equation $\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$

* It relates the electric field vector $\vec{E}$ and magnetic induction vector $\vec{B}$.
* It is a time dependent or time varying equation.
* $\vec{E}$ is generated by the time variation of $\vec{B}$.

Maxwell's Fourth Equation $\vec{\nabla}\times\vec{B} = \mu_0\left(\vec{J} + \frac{\partial\vec{D}}{\partial t}\right)$

* It is also a time dependent equation.
* It is explains Ampere's circuital Law.
* $\vec{B}$ can be produced by $\vec{J}$ and the time variation of $\vec{D}$

2. wave Equation :-

Plane Electromagnetic wave equation in Vacuum :-

Maxwell's equations in general form are,

$$\vec{\nabla}.\vec{D} = \rho \quad \ldots (1)$$

$$\vec{\nabla}.\vec{B} = 0 \quad \ldots (2)$$

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$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \quad \ldots ③$$

$$\vec{\nabla}\times\vec{H} = \vec{J}+\frac{\partial\vec{D}}{\partial t} \quad \ldots (4)$$

$\vec{J}=0$ $\quad (\because \vec{J}=\sigma\vec{E}$, and $\sigma=0)$

$\vec{\nabla}\cdot\vec{D}=0$ (or) $\vec{\nabla}\cdot\varepsilon_0\vec{E}=0$ $\quad (\because \vec{D}=\varepsilon_0\vec{E})$

$\varepsilon_0\vec{\nabla}\cdot\vec{E}=0$

$$\boxed{\vec{\nabla}\cdot\vec{E}=0} \quad \ldots (5)$$

<u>wave equation for electric field vector $(\vec{E})$</u>.

Taking the curl on both sides of equation (3).

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{E}) = \vec{\nabla}\times\left(-\frac{\partial\vec{B}}{\partial t}\right)$$

$$= -\frac{\partial}{\partial t}(\vec{\nabla}\times\vec{B})$$

$$= \partial/\partial t\,(\vec{\nabla}\times\mu_0\vec{H}) \quad (\because \vec{B}=\mu_0\vec{H})$$

(or) $\vec{\nabla}\times(\vec{\nabla}\times\vec{E}) = -\mu_0\,\partial/\partial t\,(\vec{\nabla}\times\vec{H}) \quad \ldots (6)$

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{E}) = \vec{\nabla}(\vec{\nabla}\cdot\vec{E})-\nabla^2\vec{E} \quad \ldots (7)$$

$$(\because \vec{\nabla}\cdot\vec{E}=0)$$

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{E}) = -\nabla^2\vec{E} \quad \ldots (8)$$

Substituting eqn (8) in eqn (6).

$$-\nabla^2\vec{E} = -\mu_0\,\partial/\partial t\,(\vec{\nabla}\times\vec{H})$$

Now, substituting for $\vec{\nabla}\times\vec{H}$ from eqn, (4), we get,

$$-\nabla^2\vec{E} = -\mu_0\frac{\partial}{\partial t}\left(\vec{J}+\frac{\partial\vec{D}}{\partial t}\right) \quad \ldots (9)$$

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(11)

(or) $-\nabla^2\vec{E} = -\mu_0 \frac{\partial}{\partial t}\left[\varepsilon_0 \frac{\partial \vec{E}}{\partial t}\right]$ $(\because \vec{J}=0 \ \& \ \vec{D}=\varepsilon_0\vec{E})$

$$\boxed{\nabla^2\vec{E} = \mu_0\varepsilon_0 \frac{\partial^2\vec{E}}{\partial t^2}} \quad \ldots (10)$$

This is general electromagnetic wave equation in terms of electric field vector $\vec{E}$ for free space.

<u>wave equation for magnetic field vector $(\vec{B})$.</u>

Taking curl on both sides of the equation (4).

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{H}) = \vec{\nabla}\times\left(\vec{J}+\frac{\partial\vec{D}}{\partial t}\right)$$

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{H}) = \varepsilon_0\frac{\partial}{\partial t}(\vec{\nabla}\times\vec{E}) \quad \ldots (11)$$

Now from vector calculus identity, $(\because \vec{J}=0 \ \& \ \vec{D}=\varepsilon_0\vec{E})$

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{H}) = \vec{\nabla}(\vec{\nabla}\cdot\vec{H}) - \nabla^2\vec{H} \quad \ldots (12)$$

But from eqn (2), we have,

$$\vec{\nabla}\cdot\vec{B} = 0.$$

$$\mu_0(\vec{\nabla}\cdot\vec{H}) = 0 \quad (\text{or}) \quad (\vec{\nabla}\cdot\vec{H}) = 0 \quad \ldots (13)$$

$(\because \vec{B} = \mu_0\vec{H})$

Substituting eqn (13) in eqn (12)

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{H}) = -\nabla^2\vec{H} \quad \ldots (14)$$

using eqn (14) & (11)

$$-\nabla^2 H^2 = \varepsilon_0\frac{\partial}{\partial t}(\vec{\nabla}\times\vec{E}) \quad \ldots (15)$$

Substituting the eqn (3) in eqn (15).

$$-\nabla^2\vec{H} = \varepsilon_0\frac{\partial}{\partial t}\left(-\frac{\partial\vec{B}}{\partial t}\right) \quad (\because \vec{B}=\mu_0\vec{H})$$

$$-\nabla^2\vec{H} = -\varepsilon_0\frac{\partial^2}{\partial t^2}(\mu_0\vec{H})$$

$$\text{or} \quad \nabla^2\vec{H} = \mu_0\varepsilon_0\frac{\partial^2\vec{H}}{\partial t^2} \quad \ldots (16)$$

This general electromagnetic wave equation in terms of $\vec{H}$ for free space.

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(17)

3. <u>Speed of EM wave in vacuum:-</u>

Comparing above equations (19) and (20), with the following general wave eqn ~~progar~~ propagation in x-axis,

$$\frac{\partial^2 E_x}{\partial x^2} - \mu_0 \varepsilon_0 \frac{\partial^2 E_x}{\partial t^2} = 0 \quad \ldots (19).$$

$$\frac{\partial^2 H_x}{\partial x^2} - \mu_0 \varepsilon_0 \frac{\partial^2 H_x}{\partial t^2} = 0 \quad \ldots (20).$$

$$\frac{\partial^2 y}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 y}{\partial t} = 0 \quad \ldots (21).$$

y - instantaneous displacement

c - velocity of wave.

velocity of the electromagnetic wave is given by,

$$\frac{1}{c^2} = \mu_0 \varepsilon_0$$

$$c^2 = \frac{1}{\mu_0 \varepsilon_0}.$$

Magnitude of velocity is called speed.

$$\boxed{c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}} \quad \ldots (22)$$

$\mu_0 = 4\pi \times 10^{-7}\ Hm^{-1}$, $\varepsilon_0 = 8.842 \times 10^{-12}\ Fm^{-1}$.

Substituting these values in eqn (22),

$$c = \frac{1}{\sqrt{4\pi \times 10^{-7} \times 8.842 \times 10^{-12}}}$$

$$c = 2.998 \times 10^{8}\ ms^{-1}$$

the waves in free space is a constant and equal to 3×10^8 m/s. with the velocity of light.

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wave equations for plane polarized EM wave in free space and their solution.

The electromagnetic wave eqn for $\vec{E}$ and $\vec{H}$ in free space are given by,

$$\boxed{\nabla^2 \vec{E} - \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} = 0} \quad \ldots (1)$$

$$\boxed{\nabla^2 \vec{H} - \mu_0 \varepsilon_0 \frac{\partial^2 \vec{H}}{\partial t^2} = 0} \quad \ldots (2)$$

4 conditions on the wave field:

$$E_y \neq 0, \; E_z = E_x = 0 \quad \text{and}$$

Similarly for magnetic field vector,

$$H_z \neq 0, \; H_y = H_x = 0$$

$$\nabla^2 E_y - \mu_0 \varepsilon_0 \frac{\partial^2 E_y}{\partial t^2} = 0 \quad \ldots (3)$$

$$\nabla^2 H_z - \mu_0 \varepsilon_0 \frac{\partial^2 H_z}{\partial t^2} = 0 \quad \ldots (4)$$

$$\nabla^2 E_y = \frac{\partial^2 E_y}{\partial x^2} + \frac{\partial^2 E_y}{\partial y^2} + \frac{\partial^2 E_y}{\partial z^2} \quad \ldots (5)$$

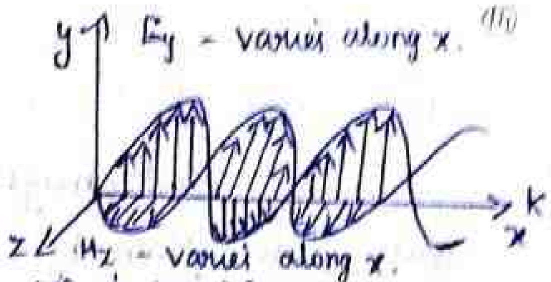
$$\cancel{\nabla^2 H_z} \; \cancel{=} \; \frac{\partial^2 H_z}{\partial x^2} = 0 \quad \text{and} \quad \frac{\partial^2 E_y}{\partial z^2} = 0.$$

$$\nabla^2 H_z = \frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + \frac{\partial^2 H_z}{\partial z^2} \quad \ldots (6)$$

similarly, $\frac{\partial^2 H_z}{\partial y^2} = 0$ and $\frac{\partial^2 H_z}{\partial z^2} = 0$.

Since at the given value of x, E_y and H_z are constant at any instant.

$$\therefore \nabla^2 E_y = \frac{\partial^2 E_y}{\partial x^2} \quad \ldots (7)$$



Similarly.

$$\nabla^2 H_z = \frac{\partial^2 H_z}{\partial x^2} \quad \ldots (8)$$

Substituting eqn (7) in eqn (5) and eqn (8) in eqn (6)

$$\frac{\partial^2 E_y}{\partial x^2} - \mu_0 \varepsilon_0 \frac{\partial^2 E_y}{\partial t^2} = 0 \quad \ldots (9)$$

$$\frac{\partial^2 H_z}{\partial x^2} - \mu_0 \varepsilon_0 \frac{\partial^2 H_z}{\partial t^2} = 0 \quad \ldots (10)$$

Solutions of the plane wave Equations:

The plane wave equations for electric field and magnetic field are given by.

$$\frac{\partial^2 E_y}{\partial x^2} - \frac{1}{c^2} \frac{\partial^2 E_y}{\partial t^2} = 0.$$

$$\frac{\partial^2 H_z}{\partial x^2} - \frac{1}{c^2} \frac{\partial^2 H_z}{\partial t^2} = 0 \qquad \left(\because \mu_0 \varepsilon_0 = \frac{1}{c^2}\right)$$

$c \rightarrow$ speed of EM wave.

The solutions of the above wave equations of progressive wave are given by.

$$E_y = E_0 \cos(\omega t - kx) \quad \ldots (11)$$

$$H_z = H_0 \cos(\omega t - kx) \quad \ldots (12)$$

where, ω - angular frequency.

k - wave vector.

The general solution of the wave equation is,

$$\vec{E_y} = E_0 e^{i(\omega t - kx)} = E_0 e^{ik(ct - x)} \quad \ldots (13)$$

$$H_z = H_0 e^{i(\omega t - kx)} = H_0 e^{ik(ct - x)} \quad \ldots (14)$$

c is the wave velocity. $(\because c = \nu\lambda)$

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5. Phase and orientation of EM wave in Matter:-

we k· It is clear that the phases of electric and magnetic fields are same. Thus both fields are in phase with each other.

Relation between electric and magnetic field vectors:-

we know for electromagnetic waves in free space.

$$\vec{E_y} = E_0 e^{ik(ct-x)} \quad \ldots (1)$$

$$\vec{H_z} = H_0 e^{ik(ct-x)} \quad \ldots (2)$$

From Maxwell's equation,

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad (\because \vec{B} = \mu_0 \vec{H})$$

$$\vec{\nabla} \times \vec{E} = -\mu_0 \frac{\partial \vec{H}}{\partial t} \qquad (\because E_x = E_z = 0)$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & E_y & 0 \end{vmatrix} = -\mu_0 \frac{\partial \vec{H}}{\partial t}$$

or $$\frac{\partial \vec{E_y}}{\partial x} = -\mu_0 \frac{\partial \vec{H_z}}{\partial t} \quad \ldots (3)$$

Substituting E_y and H_z from the eqn's (1) & (2) in eqn (3)

$$\frac{\partial}{\partial x}\left(E_0 e^{ik(\cancel{\omega}ct-x)}\right) = -\mu_0 \frac{\partial}{\partial t}\left(H_0 e^{ik(ct-x)}\right) \quad \ldots (4)$$

$$-ikE_0 e^{ik(ct-x)} = -\mu_0 (ikc) H_0 e^{ik(ct-x)}$$

$$E_0 = \mu_0 c H_0 \quad \ldots (5)$$

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \quad \ldots (6)$$

Substituting the eqn (6) in eqn (5).

$$\therefore E_0 = \mu_0 \times \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \times H_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} H_0$$

(or) $$\sqrt{\frac{\mu_0}{\varepsilon_0}} = \frac{E_0}{H_0} = \frac{E_0 e^{i(\omega t - kx)}}{H_0 e^{i(\omega t - kx)}}$$

$$\boxed{\therefore \frac{\vec{E}}{H} = \sqrt{\frac{\mu_0}{\varepsilon_0}}} \quad \ldots (7)$$

It is determined by the μ_0 and ε_0.

$$\frac{\vec{E}}{\vec{H}} = \frac{E_0}{H_0} = \sqrt{\frac{\mu_0}{\varepsilon_0}}$$

$\mu_0 = 4\pi \times 10^{-7}\ Hm^{-1}$, $\varepsilon_0 = 8.85 \times 10^{-12}\ Fm^{-1}$

$$\frac{E}{H} = \frac{E_0}{H_0} = \sqrt{\frac{\mu_0}{\varepsilon_0}} = \sqrt{\frac{4\pi \times 10^{-7}}{8.85 \times 10^{-12}}} = 376.73 = 377.$$

$$\sqrt{\frac{\mu_0}{\varepsilon_0}} = \sqrt{\frac{H/m}{F/m}} = \sqrt{\frac{\text{henry}/m}{\text{farad}/m}}$$

$$= \sqrt{\text{ohm} \times \text{ohm}} = \text{ohm}.$$

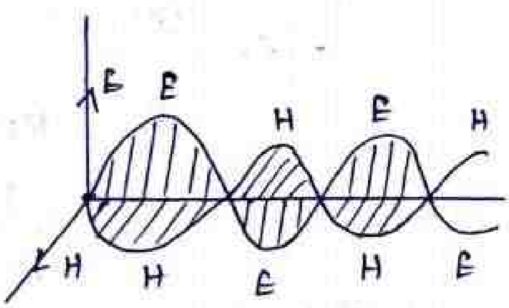
It is a constant quantity for free space and having value $= 377\,\Omega$.

poynting vector:-

The cross product of electric field vector $\vec{E}$ and the magnetic field vector $\vec{H}$ is called poynting vector. It is denoted by $\vec{S} = \vec{E} \times \vec{H}$.

$$\vec{S} = \vec{E} \times \vec{H} = \hat{j} E_y \times \hat{k} H_x$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & E_y & 0 \\ 0 & 0 & H_x \end{vmatrix} = \hat{i}\,(E_y H_x)$$



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$$S_{avg} = \frac{1}{2}(\vec{E}\times\vec{H}) = \frac{1}{2}E_0 \times H_0 = \frac{E_0}{\sqrt{2}} \times \frac{H_0}{\sqrt{2}} = E_{rms} \cdot H_{rms}$$

$$\left(\because E_{rms} = \frac{E_0}{\sqrt{2}} \text{ and } H_{rms} = \frac{H_0}{\sqrt{2}}\right)$$

Maxwell's findings is summarised as below:-

* If there is a varying electric field in vacuum, there is also a varying magnetic field and vice versa.
* The electric and magnetic fields obey wave equation with identical propagation speeds.
* The speed of propagation given by $c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$ is the same as the measured speed of light.
* The light waves can, therefore, be identified as electromagnetic waves.
* The electric and magnetic fields in electromagnetic waves oscillate in phase with each other.

6. Propagation of electromagnetic wave through A dielectric Medium. (Non-conducting isotropic Medium).

Maxwell's equations are,

$$\vec{\nabla}\cdot\vec{D} = \rho$$

$$\vec{\nabla}\cdot\vec{B} = 0$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla}\times\vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

In an isotropic dielectric,

$$\vec{D} = \epsilon\vec{E},\ B = \mu\vec{H},\ \vec{J} = \sigma\vec{E} = 0 \text{ and } \rho = 0.$$

Therefore, Maxwell's eqn in this case take the form,

$$\vec{\nabla}\cdot\vec{E} = 0 \quad \ldots (1)$$

$$\vec{\nabla}\cdot\vec{H} = 0 \quad \ldots (2)$$

$$\vec{\nabla}\times\vec{E} = -\mu\frac{\partial \vec{H}}{\partial t} \quad \ldots (3)$$

$$\vec{\nabla}\times\vec{H} = \epsilon\frac{\partial \vec{E}}{\partial t} \quad \ldots (4)$$

Equation of propagation of magnetic vector, H.

Taking curl of eqn. (4),

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{H}) = \epsilon\vec{\nabla}\times\left(\frac{\partial \vec{E}}{\partial t}\right)$$

$$\vec{\nabla}(\vec{\nabla}\cdot\vec{H}) - \nabla^2\vec{H} = \epsilon\frac{\partial}{\partial t}(\vec{\nabla}\times\vec{E}) \quad \ldots (5)$$

putting values from eqns (2) and (3).

$$\nabla^2\vec{H} = \mu\epsilon\frac{\partial^2\vec{H}}{\partial t^2} \quad \ldots (6)$$

Equation of propagation of electric vector, E.

Taking of eqn (3),

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{E}) = \vec{\nabla}\times\left(-\mu\frac{\partial \vec{H}}{\partial t}\right)$$

$$\vec{\nabla}(\vec{\nabla}\cdot\vec{E}) - \nabla^2\vec{E} = -\mu\frac{\partial}{\partial t}(\vec{\nabla}\times\vec{H})$$

putting values from eqns (1) & (4) we get,

$$\nabla^2\vec{E} = \mu\epsilon\frac{\partial^2\vec{E}}{\partial t^2} \quad \ldots (7).$$

The eqns (6) and (7) is compared with the general wave eqn.

$$\nabla^2 u = \frac{1}{v^2}\frac{\partial^2 u}{\partial t^2}.$$

where v is the speed of wave.

$$v = \frac{1}{\sqrt{\mu\epsilon}}$$

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Therefore, Maxwe $\frac{1}{\sqrt{\mu_0 \varepsilon_0}} = c$.

Refractive index is.

$$n = \frac{c}{v} = \sqrt{\frac{\mu \varepsilon}{\mu_0 \varepsilon_0}} = \sqrt{\mu_r \varepsilon_r}$$

In a non-magnetic medium $\mu_r = 1$

$$\boxed{\therefore\ n = \sqrt{\varepsilon_r}}$$

---

7. EM waves in conducting Medium:-

General Maxwell's equations are

$$\vec{\nabla} \cdot \vec{D} = \rho \quad \dots \quad (1)$$

$$\vec{\nabla} \cdot \vec{B} = 0 \quad \dots \quad (2)$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \dots \quad (3)$$

$$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \quad \dots \quad (4)$$

conducting medium $\sigma \neq 0$, $\rho = 0$.

therefore the eqn (1) reduces to $\vec{\nabla} \cdot \vec{D} = 0$

(or) $\vec{\nabla} \cdot (\varepsilon \vec{E}) = 0 \Rightarrow \vec{\nabla} \cdot \vec{E} = 0 \quad \dots \quad (5)$

'ε' is permittivity of the medium. $(\vec{D} = \varepsilon \vec{E})$

Taking the curl on both sides of eqn. (3).

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = \vec{\nabla} \times \left[-\frac{\partial \vec{B}}{\partial t}\right] \quad \dots \quad (6)$$

From vector calculus identity.

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{E}) - \nabla^2 \vec{E} \quad \dots \quad (7)$$

But from eqn (5) $\vec{\nabla} \cdot \vec{E} = 0$

therefore, eqn (7) becomes,

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = -\nabla^2 \vec{E} \quad \dots \quad (8)$$

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$$\vec{\nabla} \times \left[-\frac{\partial \vec{B}}{\partial t}\right] = -\mu \frac{\partial}{\partial t}(\vec{\nabla} \times \vec{H}) \quad \ldots (9). \qquad (\because \vec{B} = \mu H)$$

substituting the eqn (8) & (9) in (6).

$$\cancel{-}\nabla^2 \vec{E} = \cancel{-}\mu \frac{\partial}{\partial t}(\vec{\nabla} \times \vec{H}) \quad \ldots (10).$$

on substituting the value of $(\vec{\nabla} \times \vec{H})$ from eqn. (4) in eqn (10).

$$\nabla^2 \vec{E} = \mu \frac{\partial}{\partial t}\left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right) \quad \ldots (11)$$

$\vec{J} = \sigma \vec{E}$ and $\vec{D} = \varepsilon \vec{E}$ eqn (11) becomes.

$$\nabla^2 \vec{E} = \mu\sigma \frac{\partial \vec{E}}{\partial t} + \mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\boxed{\nabla^2 \vec{E} - \mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2} - \mu\sigma \frac{\partial \vec{E}}{\partial t} = 0} \quad \ldots (12).$$

$$\boxed{\nabla^2 \vec{H} - \mu\varepsilon \frac{\partial^2 \vec{H}}{\partial t^2} - \mu\sigma \frac{\partial \vec{H}}{\partial t} = 0} \quad \ldots (13).$$

<u>wave eqn for plane polarized EM waves:-</u>

$$\boxed{\frac{\partial^2 E_y}{\partial x^2} - \mu\varepsilon \frac{\partial^2 E_y}{\partial t^2} - \mu\sigma \frac{\partial E_y}{\partial t^2} = 0} \quad \ldots (14).$$

$$\boxed{\frac{\partial^2 H_z}{\partial x^2} - \mu\varepsilon \frac{\partial^2 H_z}{\partial t^2} - \mu\sigma \frac{\partial H_z}{\partial t} = 0} \quad \ldots (15)$$

$\therefore \mu\varepsilon = \frac{1}{v^2}$.

the product $\mu\sigma$ is called magnetic diffusivity.

$$\boxed{\vec{E_y} = E_0 e^{-kx} e^{i(\omega t - kx)}} \quad \ldots (16).$$

This is a progressive wave having amplitude equal to $E_0 e^{-kx}$.

Electromagnetic waves:-

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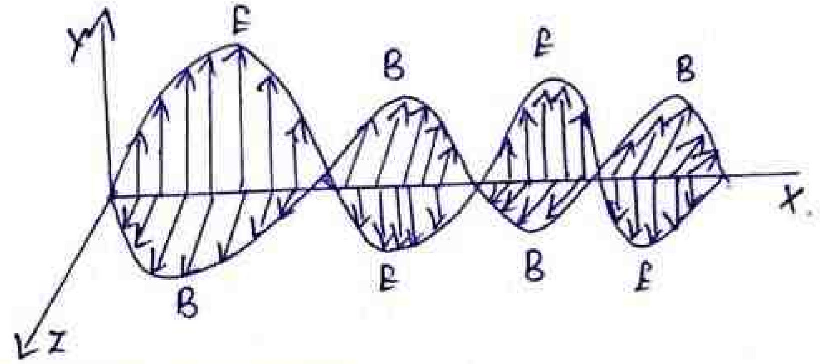
* According to Maxwell, an accelerated charge is a source of electromagnetic radiation.

* In an electromagnetic wave, electric and magnetic field vectors are at right angles to each other and both are at right angles to the direction of propagation.

* They possess the wave character and propagate through free space without any material medium.

* These waves are transverse in nature.

* The variation of electric field $\vec{E}$ along y direction and magnetic field $\vec{B}$ along z direction and wave propagation in x direction.



8. properties of Electromagnetic waves:-

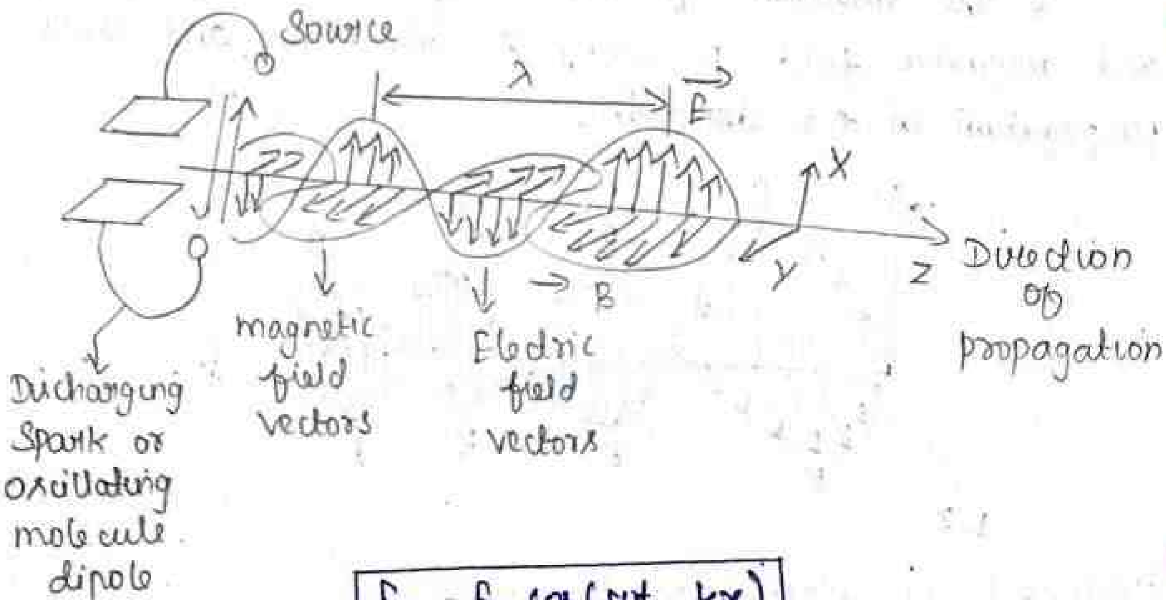
* Electromagnetic waves are produced by accelerated charges.
* they do not require any material for propagation.
* variation of maxima and minima in both $\vec{E}$ and $\vec{B}$ occur simultaneously.
* They travel in vacuum or free space with a speed $3 \times 10^8 \, ms^{-1}$ given by the relation $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$

μ_0 - permeability of free space,
ε_0 - permittivity of free space.

* The energy in an electromagnetic wave is equally divided between electric and magnetic field vectors.
* The electromagnetic waves being chargeless, they are not deflected by electric and magnetic fields.

9. Localized Sources for Electromagnetic waves:-

* Any stationary charge produces only electric field.
* If the charged particle accelerates, it produces magnetic field in addition to electric field.



$$\boxed{E_y = E_0 \cos(\omega t - kx)}$$

$$\boxed{B_z = B_0 \cos(\omega t - kx)}$$

* where E_0 and B_0 are amplitudes of oscillating electric and magnetic field.
* k is a wave number denotes the direction of propagation of electromagnetic wave.
* ω is the angular frequency of the wave.

In free space or in vacuum,

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$$\boxed{c = \frac{E_0}{B_0}}$$

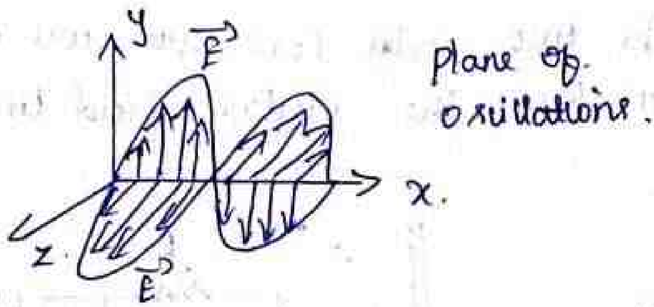
In any medium,

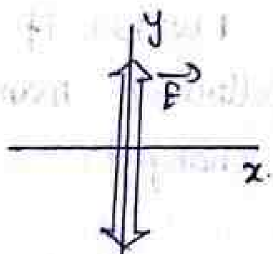
$$\boxed{v = \frac{E_0}{B_0} < c}$$

c is the speed of light.

10. Polarization:-

* In an electromagnetic wave, the direction in which the electric field's amplitude vector $\vec{E_0}$ points, specifies the geometrical orientation of the oscillation.
* The direction of $\vec{E_0}$ is now called the polarization of the wave.
* It is continuously changing between directed up and down along the y-axis.





This is because as Gauss law for electric field for a a system with a vanishing charge density.

$$\vec{\nabla}\cdot\vec{E} = 0 \quad \dots (1)$$

$$\frac{\partial E_x}{\partial x} = 0, \quad \frac{\partial E_y}{\partial y} = 0, \quad \frac{\partial E_z}{\partial z} = 0 \quad \dots (2)$$

$\vec{E}(x,t) = \vec{E_0}\cos(\omega t - kx)$ cannot have an oscillating amplitude in the x-direction.. $\frac{\partial E_x}{\partial x} = 0$. So EM waves are never longitudinally polarized.

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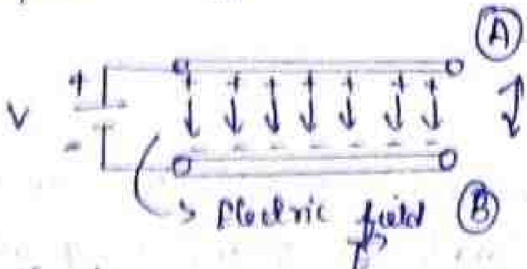
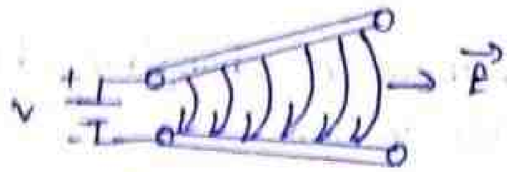
11. Producing Electromagnetic waves:-

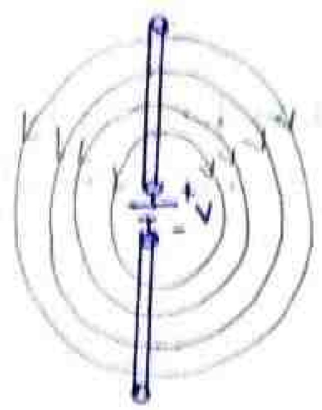
Let us consider a capacitor like arrangement as shown in fig. here A and B are two conducting rods. They are separated by a distance 'd.'

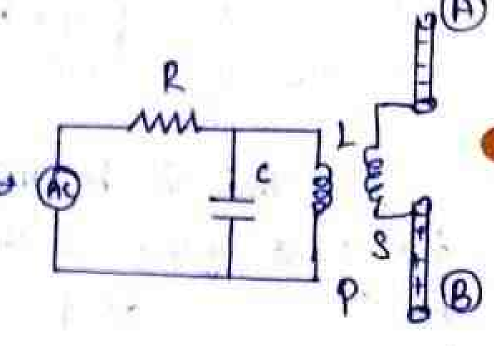


If the two rods get tilted in the following manner correspondingly the electric field pattern also changes.



Further if the two rods (or) wires are tilted in the following manner then, the electric field lines are also changed.

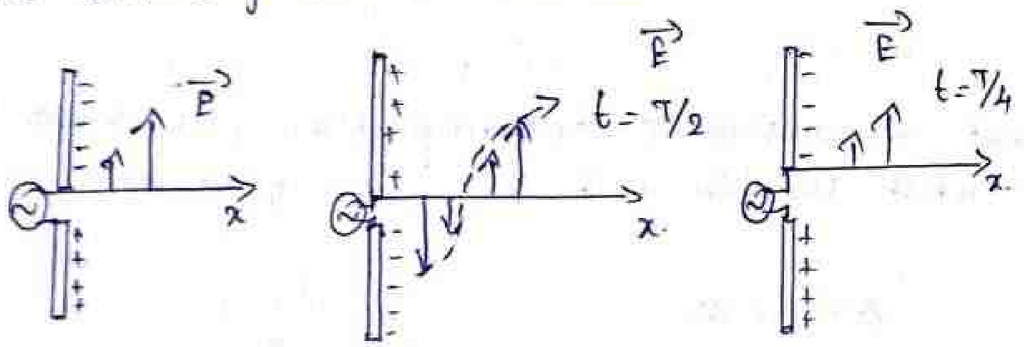


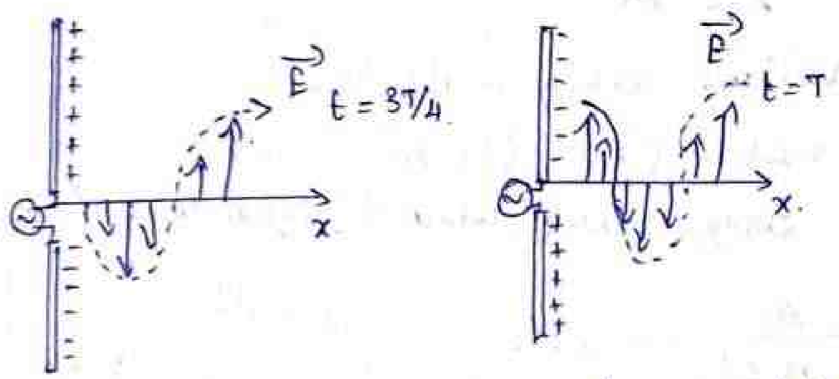


Now by replacing the voltage source as a AC voltage source. then this arrangement is called as dipole antenna.

A basic design of a electromagnetic wave generator involves this dipole Antenna arrangement.

The secondary of the transformer is attached to the two conducting rods (A) and (B).

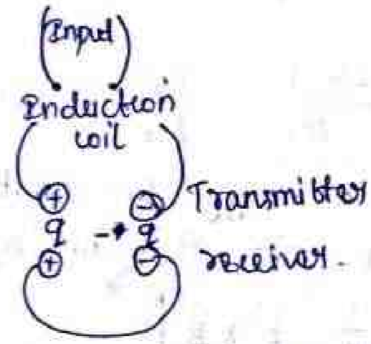




Most of the dipole antennas have a total length of $\lambda/2$ here λ is the wavelength of EM.

<u>production of electromagnetic waves - Hertz experiment</u>:

Maxwell's prediction was experimentally confirmed by Heinrich Rudolf Hertz in 1888. The experimental set up used is fig.



Since the coil is maintained at very high potential, and air between the electrodes gets ionized and spark is produced.

If the receiver is rotated by 90°, then no spark is observed by the receiver.

Hertz detected radio waves and also computed the speed of radio waves which is equal to the speed of light. (26)

12. Electromagnetic Energy flow and poynting vector:-

Let us consider a stationary plane perpendicular to the x-axis which coincides with the wave front at a certain time t.

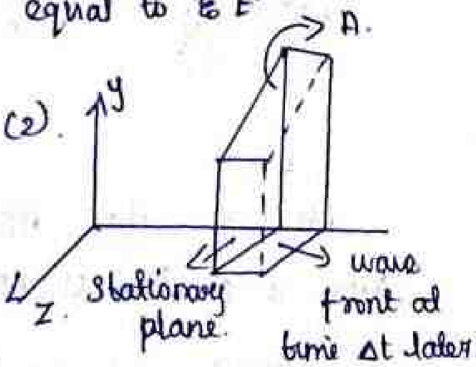
$$\Delta x = c\Delta t \qquad \left(c = \frac{\Delta x}{\Delta t}\right)$$

$$\Delta V = A.\Delta x = A.c.\Delta t$$

If ΔU is the available energy in this volume,

$$\Delta U = u\Delta V = (\varepsilon_0 E^2)(Ac\,\Delta t) \quad \ldots (1)$$

here 'u' is the energy density which is equal to $\varepsilon_0 E^2$



$$S = \frac{\Delta U}{A.\Delta t} = \varepsilon_0 E^2 c \quad \ldots (2)$$

$$E = cB \quad \& \quad c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$$

eqn (2) becomes.

$$S = \varepsilon_0 c^2 B^2 c \quad \ldots (3)$$

$$S = \frac{\varepsilon_0 c B^2}{\varepsilon_0 \mu_0} \quad \ldots (4) \qquad \left(\because c^2 = \frac{1}{\varepsilon_0 \mu_0}\right.$$

$$S = \frac{cB.B}{\mu_0} = \frac{EB}{\mu_0} \quad \ldots (5) \qquad \left.\because c\vec{B} = \vec{E}\right)$$

The unit of 'S' is energy per unit area per unit time or power per unit area. The SI unit of 'S' is W/m^2.

$$\boxed{\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}} \quad \ldots (6) \qquad \left(\vec{S} = \vec{E} \times \frac{\vec{B}}{\mu_0}\right.$$

$$\boxed{\vec{S} = \vec{E} \times \vec{H}} \quad \ldots (7) \qquad \left.\because \vec{B} = \mu_0 \vec{H}\right)$$

eqn (6) is the poynting vector in vacuum.

13. Intensity of AN EM wave in vacuum:-

Let us consider the electric and magnetic field solutions. $\vec{E}(x,t) = E_y \cos(\omega t - kx)$ ... (8)

$\vec{B}(x,t) = B_z \cos(\omega t - kx)$ ...

from eqn (6).

$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$$

$$\vec{S}(x,t) = \frac{1}{\mu_0} E_y \cos(\omega t - kx) \times B_z \cos(\omega t - kx) \quad \ldots (9)$$

the x-component of the poynting vector is given by.

$$S_x(x,t) = \frac{E_y B_z}{\mu_0} \cos^2(\omega t - kx) \quad \ldots (10)$$

$$= \frac{E_y B_z}{\mu_0} \left(\frac{1 + \cos 2(\omega t - kx)}{2} \right) \quad \ldots (11)$$

$$S_{average} = \bar{S}_x(x,t) \; \frac{E_x B_y}{2\mu_0} \quad \ldots (12)$$

$$S_{av} = \frac{E_y B_z}{2\mu_0} = \frac{E_y \cdot E_y}{2\mu_0 c} \quad \ldots (13)$$

$$= \frac{E_y E_y}{2\mu_0 \times \frac{1}{\sqrt{\mu_0 \varepsilon_0}}} \Rightarrow S_{av} = \frac{1}{2}\sqrt{\frac{\varepsilon_0}{\mu_0}} E_y^2$$

$$S_{av} = \frac{\varepsilon_0}{\sqrt{\mu_0 \varepsilon_0}} E_y^2$$

$$\boxed{I = S_{av} = \frac{1}{2} \varepsilon_0 c E_y^2}$$

This is the intensity of an EM wave in vacuum.

Also intensity is represented as for localised sources as

$$I = \frac{\text{Power}}{\text{Area}} = \frac{P}{4\pi r^2}$$

14. Momentum And Radiation Pressure:-

Maxwell proved that wave energy U and momentum are related by. $p = \frac{U}{c}$ .... (1).

where U is energy density and c is the velocity of light.

As the electromagnetic waves carry momentum, they exert pressure, when they are reflected or absorbed at the surface of a body. This is known as radiation pressure.

From newton's second Law,

$$F = \frac{\Delta p}{\Delta t} \quad \ldots (2).$$

As intensity $I = \frac{\text{power}}{\text{Area}} = \frac{\text{energy / time}}{\text{Area}}$.

$$\Delta U = I . A . \Delta t \quad \ldots (3).$$

from eqn (1) the momentum is,

$$\Delta p = \frac{\Delta U}{c} = \frac{I . A . \Delta t}{c} \quad \ldots (4).$$

$$F = \frac{\Delta p}{\Delta t} = \frac{I . A}{c} \quad \ldots (5).$$

$$F = \frac{2IA}{c} \quad \ldots (6).$$

Radiation pressure:-

The force per unit area on an object due to EM radiation is the radiation pressure P_r.

Thus from eqns. (5) and (6) we obtain,

$$\text{Radiation pressure } P_r = \frac{F}{A}$$

$$P_r = \frac{I}{c}$$

for total absorption of radiation.

$$\boxed{P_r = \frac{2I}{c}}$$

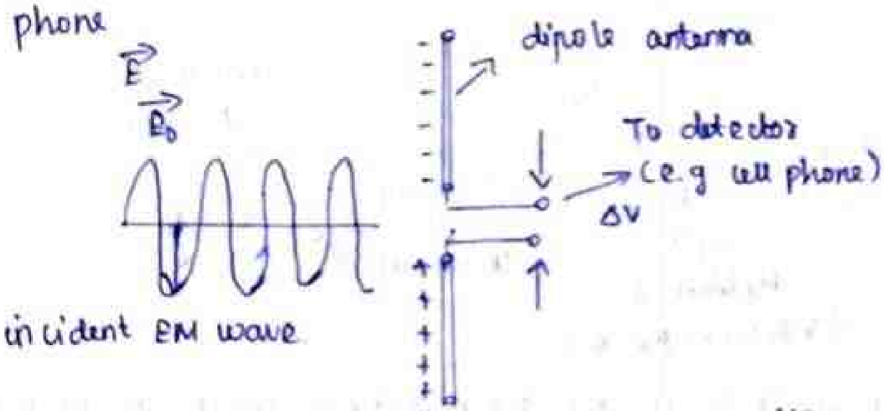
for total reflection back along the path.

15. Cell phone Reception:-

* A typical cell phone contains a tiny low-power radio transmitter or antenna.

* The antenna's length is comparable to $\lambda/2$, where 'λ' is the wavelength of the EM signal being emitted by the cell phone.

* As λ is short, so the cell phone antenna is also very short. Typically a simple dipole antenna as shown in the fig. is used to detect the incoming EM signal in the cell phone



* This induced voltage is then amplified and processed by circuitory in the cell phone.

* The low power signals emitted by the cell phone will be received and transmitted by the cell phone towers. The two towers are also another type of antenna.

* The cell phone transmits on one frequency and receive with other frequency.

16. Reflection and Transmission of EM waves vaccum-Non-conducting Medium interface for Normal incidence.

Let us consider a monochromatic uniform plane wave that travels through one medium and enters another medium of infinite extent.

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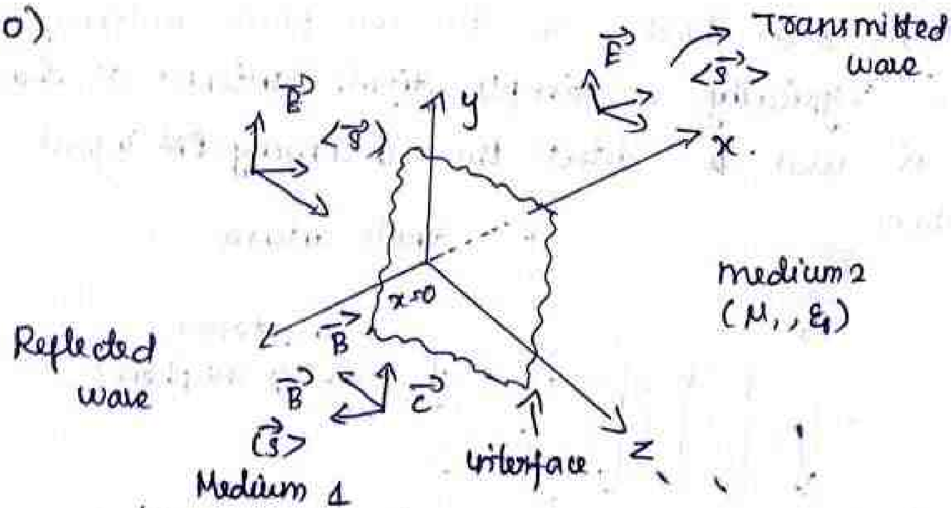
The uniform plane EM wave propagating along x-direction in a vaccum medium (μ_0, ε_0) incident normally on the surface of a flat non-conducting medium permittivity.

The region to the left of the interface is medium 1 $(x \leq 0)$ and region to the right of the interface is medium 2 $(x \geq 0)$.



If $(k_1 = \omega / v_1)$ is the propagation constant of this wave in medium-1. Then the electric and magnetic field waves are represented as.

$$\vec{E_i}(x,t) = E_0 \cos(\omega t - k_1 x) \quad \ldots (1)$$

$$\vec{B_i}(x,t) = B_0 \cos(\omega t - k_1 x) \quad \ldots (2) \qquad \left(\because B_0 = \frac{E_0}{V_1}\right)$$

Then, the reflected waves are represented as,

$$\vec{E_R}(x,t) = E_1 \cos(\omega t + k_1 x) \quad \ldots (3)$$

$$\vec{B_R}(x,t) = \frac{E_1}{V_1} \cos(\omega t + k_1 x) \quad \ldots (4)$$

$$\vec{E_T}(x,t) = E_2 \cos(\omega t - k_2 x) \quad \ldots (5)$$

$E_0 = cB_0$

$$\vec{B_T}(x,t) = \frac{E_2}{v_2} \cos(\omega t - k_2 x)$$

Here in eqns (3) and (4), the sign is reversed used on the wave number k to denote that this wave is propagation.

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Also the wave numbers k_1 and k_2 are related to.

$$k_1 = \frac{\omega}{v_1} \quad \ldots (7)$$

$$k_2 = \frac{\omega}{v_2} \quad \ldots (8)$$

where v_1 and v_2 are the velocities of EM waves in medium 1 and medium 2 respectively.

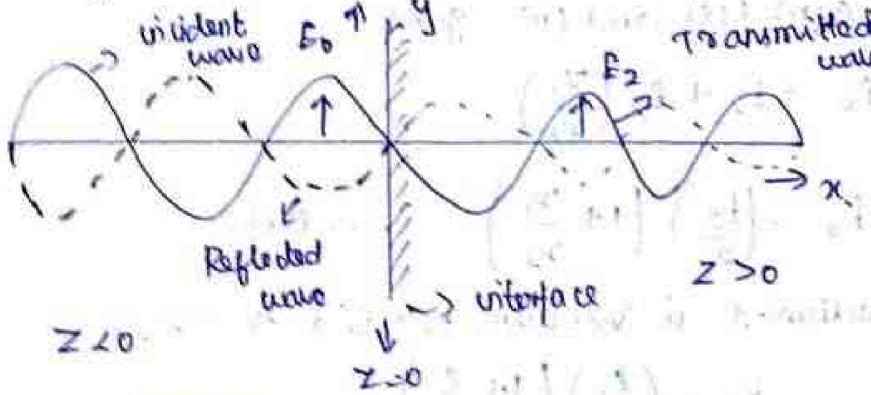
$$\vec{E_y}(x,t) = E_0 \cos(\omega t - k_1 x) + E_1 \cos(\omega t + k_1 x) \ldots (9)$$

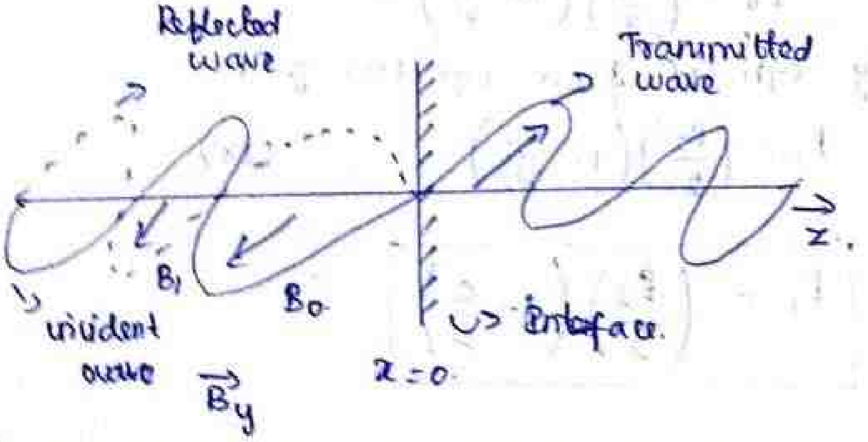
(or)

$$\vec{E_y}(x,t) = \vec{E_i}(x,t) + \vec{E_R}(x,t) \quad \ldots (10)$$

The total instantaneous electric field $\vec{E_y}$ for any value of x in the medium - 2 is,

$$\vec{E_y}(x,t) = E_2 \cos(\omega t - k_2 x) \quad \ldots (11)$$





$$E_0 \cos(\omega t - k_1 x) + E_1 \cos(\omega t - k_1 x) = E_2 \cos(\omega t - k_2 x)$$

$x = 0$, $\quad E_0 \cos(\omega t) + E_1 \cos(\omega t) = E_2 \cos(\omega t) \quad \ldots (12)$

(or)

$$E_0 + E_1 = E_2 \quad \ldots (13)$$

$x = 0$,

$$\frac{dE_i}{dx} + \frac{dE_R}{dx} = \frac{dE_T}{dx} \quad \ldots (14)$$

$$-E_0 k_1 \sin(\omega t) - E_1 k_1 \sin(\omega t) = E_2 k_2 \sin(\omega t) \quad \ldots (15)$$

(or) $\quad E_0 k_1 - E_1 k_1 = E_2 k_2$

(or) $\quad k_1 (E_0 - E_1) = E_2 k_2$.

(or) $\quad E_0 - E_1 = E_2 \cdot \left(\frac{k_2}{k_1}\right) \quad \ldots (16)$.

$k_1 = \frac{\omega}{v_1}$ and $k_2 = \frac{\omega}{v_2}$ then eqn, (16)

$$E_0 - E_1 = E_2 \cdot \left(\frac{v_1}{v_2}\right) \quad \ldots (17)$$

Adding eqns (13) and (17) gives,

$$2E_0 = E_2 + E_2 \left(\frac{v_1}{v_2}\right)$$

$$E_0 = \left(\frac{E_2}{2}\right)\left(1 + \frac{v_1}{v_2}\right) \quad \ldots (18)$$

when medium-1 is vacuum $v_1 = c$, & $v_2 = v$

$$\therefore E_0 = \left(\frac{E_2}{2}\right)\left(1 + \frac{c}{v}\right)$$

Subtracting eqn (17) from eqn (13) gives.

$$E_1 = \left(\frac{E_2}{2}\right)\left(1 - \frac{v_1}{v_2}\right) \quad \ldots (19)$$

$$\boxed{E_1 = \left(\frac{E_2}{2}\right)\left(1 - \frac{c}{v}\right)}$$

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(1)

Unit-III

Oscillations, optics and Lasers.

1. Simple Harmonic Motion:-

An oscillatory motion is harmonic if the displacement can be expressed in terms of sine or cosine function. An oscillator executing harmonic motion is called a harmonic oscillator.

Definition:-

When the acceleration of particle is directly proportional to its displacement from its equilibrium position and its always directed towards equilibrium position, then the motion of the particle is said to be simple harmonic motion.

Characteristics of simple harmonic motion:-

- A particle executing simple harmonic motion must satisfy the following conditions.
- The motion must be periodic
- The motion is oscillator i.e, to and fro along a straight line or along a curved path about a mean position.
- If there is no air resistance or friction, the motion once started will continue indefinitely.

Example for simple harmonic motion:-

Simple pendulum:-

It consists of an ideally massless inextensible string hanging from a rigid support 'A' with a point mass connected to its other end 'B'.

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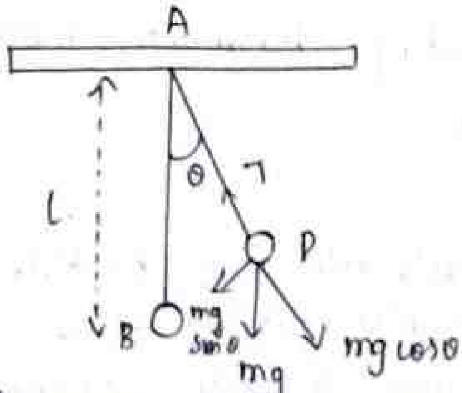
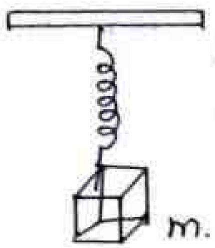
If θ is the angle made by the string at P with the equilibrium condition of the string AB, m and g are the mass of the bob and the acceleration due to gravity. then the radial component of mg balances the tension T across the string.



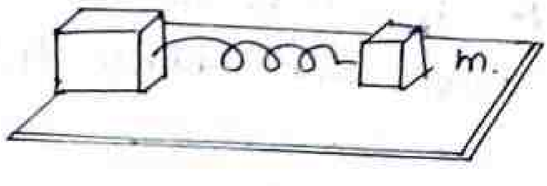
The tangential component 'mg sin θ' tries to bring the bob at B. There way oscillations continue till it stops due to air friction.

Spring - Mass System:-

Here a mass black is connected to a spring either vertically or horizontally from rigid supports as in fig. on displacing the mass from its equilibrium position and then releasing it, again simple harmonic oscillators sol is.



vertically connected



Horizontally connected.

- Vibrations of a tuning fork.
- vibrations of a Sonometer wire.
- vertical oscillations of the liquid column in a U-Tube.
- Angular oscillations of a torsion pendulum.

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Types of Simple Harmonic Motion :-

The simple harmonic motions are of two types.

i) Linear Simple Harmonic Motion :-

If the displacement of a particle executing S.H.M is linear, the motion is said to be linear S.H.M.

The examples of linear S.H.M are motion of simple pendulum, the motion of prongs of vibrating tunning fork the motion of a point mass attached to a spring.

ii) Angular Simple Harmonic Motion :-

If the displacement of a particle executing S.H.M is angular, the motion is said to be angular S.H.M.

The (eg) of angular S.H.M. is torsional oscillations of a solid.

Essential conditions for S.H.M :-

If a is linear acceleration and y is displacement from equilibrium position, then essential condition for linear S.H.M is,

$$a \propto -y.$$

α is a angular acceleration, and θ angular displacement from equilibrium position, then essential condition for angular S.H.M is,

$$\alpha \propto -\theta.$$

Definitions concerning S.H.M.

Let a particle execute S.H.M along straight line QOP, about O.

④

1. Amplitude :-

The maximum displacement of a particle from mean position is called the amplitude. It is denoted by A. Then

$$OP = OQ = A.$$

2. Oscillation :-

When particle moves from mean position O to P returns from P to Q via O and then comes back from Q to O;

then particle is said to complete one-oscillation

1 oscillation = motion from O to P + from P to Q + from Q to O or motion.

3. Period :-

The time taken by the particle executing S.H.M to complete one oscillation is called the period or periodic time. It is denoted by T.

4. Frequency :-

The number of oscillations completed by particle in one second is called is frequency. It is denoted by n.

$$\text{Frequency } n = \frac{1}{\text{period } (T)}.$$

5. Phase :-

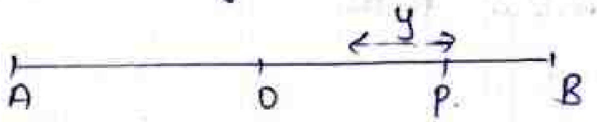
The position and direction of motion of a vibrating particle is different at different instants.

The instantaneous position and direction of motion of a vibrating particle is expressed by a physical quantity called the phase.

If S.H.M is expressed as $y = A\sin(\omega t + \phi)$; then the quantity $(\omega t + \phi)$ is the phase of vibrating particle.

Differential Equation of S.H.M.

A particle executing S.H.M is called a harmonic oscillator, let y be the displacement of particle from mean position at any time t.



$$F \propto -y$$

or $F = -ky \quad \ldots (1)$

where k is a constant of proportionality and it is called spring factor or force constant. Its unit is newton/metre (N/m)

If $a = \frac{d^2y}{dt^2}$ is acceleration at any instant t, then by Newton's second law of motion F = mass × acceleration

$= ma.$

$$F = m\frac{d^2y}{dt^2} \quad \ldots (2)$$

From the eqn (1) & (2),

$$m\frac{d^2y}{dt^2} = -ky.$$

$$\therefore m\frac{d^2y}{dt^2} + ky = 0.$$

$$\frac{d^2y}{dt^2} + \frac{k}{m}y = 0.$$

$$\frac{d^2y}{dt^2} + \omega^2 y = 0 \quad \ldots (3).$$

where $\omega^2 = \frac{k}{m}$ is a constant and ω is known as angular frequency.

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(6)

The eqn (3) represents the differential eqn of SHM

A general solution of the differential eqn for S.H.M is given by.

$$\boxed{y = A\sin(\omega t + \phi)} \quad \text{- - - - (4).}$$

where, A is the amplitude of the S.H.M

ϕ is the initial phase.

Angular harmonic motion:-

Now consider a particle executing angular harmonic motion. At any instant t, let θ be the angular displacement measured from the equilibrium position of the particle. Then Similar to eqn (3) for the linear case,

$$\frac{d^2\theta}{dt^2} + \omega^2\theta = 0 \quad \text{- - - - (5)}$$

ω^2 is a constant.

Velocity and Acceleration:-

we have displacement $y = A\sin(\omega t + \phi)$

Differentiating with respect to time t.

$$\frac{dy}{dt} = v = A\omega\cos(\omega t + \phi) \quad \text{- - - (6).}$$

or) $\cos^2(\omega t + \phi) = 1 - \sin^2(\omega t + \phi)$

$$v = A\omega\left(\sqrt{1 - \sin^2(\omega t + \theta)}\right)$$

$(\because \sin^2(\omega t + \phi) + \cos^2(\omega t + \phi) = 1$

$\cos^2(\omega t + \phi) = 1 - \sin^2(\omega t + \phi)$)

$$v = \omega\sqrt{A^2 - A^2\sin^2(\omega t + \phi)}$$

(or)

$$v = \omega\sqrt{A^2 - y^2} \quad \text{- - - - - (7).}$$

$$\therefore \boxed{v_{max} = \omega A}$$

7

Acceleration:-

Differentiating eqn (6) with respect to time t,

acceleration $a = \frac{dv}{dt} = -A\omega^2 \sin(\omega t + \phi)$

or $\boxed{a = -\omega^2 y}$ ...... (8)

This eqn is the standard eqn of S.H.M.

For maximum acceleration at $y = A$

$\therefore$ maximum acceleration $a_{max} = \omega^2 A$.

minimum acceleration is obtained by putting $y = 0$.

$\therefore$ $a_{min} = 0$.

Period of S.H.M:-

The time taken by the particle to make one complete to and fro motion is called the time period of the S.H.M. Since ω is the uniform angular velocity.

$$\omega = \frac{2\pi}{T}$$

or period $T = \frac{2\pi}{\omega}$ ..... (9)

From eqn (8), we have,

$$\omega^2 = \frac{a}{y}$$

$$\omega = \sqrt{a/y}$$

(or) $\frac{1}{\omega} = \sqrt{y/a}$.

Substituting this in eqn (9).

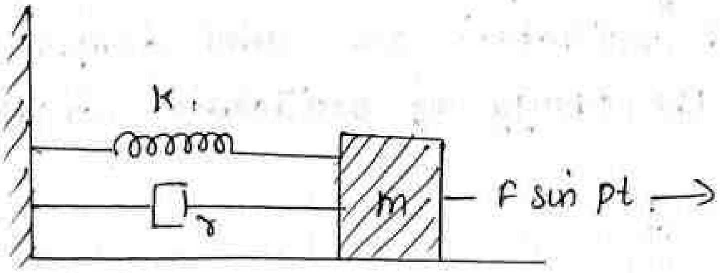
Time period $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{y}{a}}$ ..... (10)

$$T = 2\pi\sqrt{\frac{\text{displacement}}{\text{Acceleration}}}$$

(16)

Differential equation for forced oscillations :-

consider a particle of mass m connected to a spring. This particle is driven by a periodic force.



Mechanical forced oscillator with force F sin pt.

The oscillations are started and the forces acting on the particles are

i) a restoring force :-

It is proportional to the displacement acting in the opposite direction. It is given by $-ky$ where k is known as the restoring force constant.

ii) a frictional force :-

It is proportional to velocity but acting on the opposite direction. It is given by $-r\frac{dy}{dt}$ where r is the frictional force constant.

iii) the external periodic force :-

$F \sin pt$ where F is the maximum value of the force and p is the angular frequency.

therefore, net force F' acting on the particle

$$F' = -ky - r\frac{dy}{dt} + F \sin pt \quad \cdots\cdots (1)$$

By newton's second law of motion, the resultant force acting on the particle

$F' =$ mass × acceleration $= ma$.

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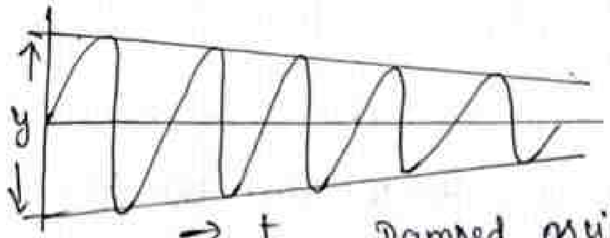
b) Damped oscillation:-

Most of the oscillation occur in air or in a medium. Hence the medium offer some resistive force on the oscillating body.

Such oscillations are called damped oscillations, As a result the energy of oscillations decreases with time.



Damped oscillation.

Characteristic of a damped oscillation:-

* Amplitude of oscillation is not a constant.
* There is dissipation of energy
* Small changes are produced in the frequency of oscillation. eg. The oscillations of a pendulum in air.

e. Forced vibration:-

When a body A is maintained in the state of vibration by a periodic force of frequency ν other than its natural frequency (ν_0) of the body. The vibrations are called forced vibrations.

Characteristics of forced oscillations are:-

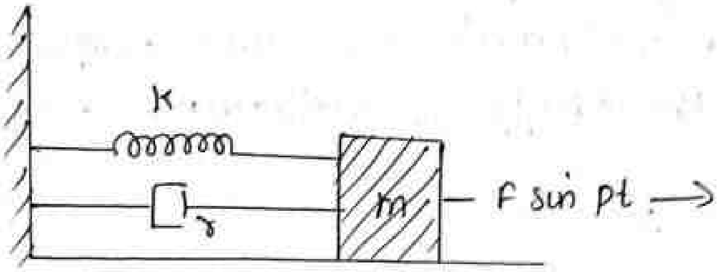
Amplitude will be a constant.

The frequency of forced vibration is equal to that of the external periodic force.

eg:- press the stem of a vibration of tuning fork, against table, the table suffer forced vibrations.

Differential equation for forced oscillations:

consider a particle of mass m connected to a spring. This particle is driven by a periodic force.



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iii) the external periodic force:-

$F \sin pt$ where F is the maximum value of the force and p is the angular frequency.

Therefore, net force F' acting on the particle.

$$F' = -ky - r\frac{dy}{dt} + F\sin pt \quad \cdots\cdots (1)$$

By newton's second law of motion, the resultant force acting on the particle.

$$F' = \text{mass} \times \text{acceleration} = ma.$$

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$$F' = m\frac{d^2y}{dt^2} \quad \ldots (2) \qquad \left(\because a = \frac{d^2y}{dt^2}\right)$$

$\therefore$ From the eqn (1) & (2),

$$m\frac{d^2y}{dt^2} = -ky - r\frac{dy}{dt} + F\sin pt.$$

or,
$$m\frac{d^2y}{dt^2} + r\frac{dy}{dt} + ky = F\sin pt \quad \ldots (3).$$

$$\frac{\cancel{m}}{\cancel{m}}\frac{d^2y}{dt^2} + \frac{r}{m}\frac{dy}{dt} + \frac{k}{m}y = \frac{F}{m}\sin pt.$$

$$\frac{d^2y}{dt^2} + 2b\frac{dy}{dt} + \omega^2 y = f\sin pt \quad \ldots (4).$$

where $\frac{r}{m} = 2b$, $\frac{k}{m} = \omega^2$ and $\frac{F}{m} = f$

The eqn (4) is the differential equation of the motion of the forced oscillation of the particle.

The solution of differential eq. (4)

$$y = A\sin(pt - \theta) \quad \ldots (5)$$

where A is the steady amplitude of vibration,

we have,
$$A = \frac{f}{\sqrt{[(\omega^2 - p^2)^2 + 4b^2p^2]}} \quad \ldots (6)$$

$$\tan\theta = \frac{2bp}{\omega^2 - p^2}$$

or,
$$\theta = \tan^{-1}\left(\frac{2bp}{\omega^2 - p^2}\right) \quad \ldots (7)$$

The equation (6) gives the amplitude of forced vibration while eqn (7) its phase.

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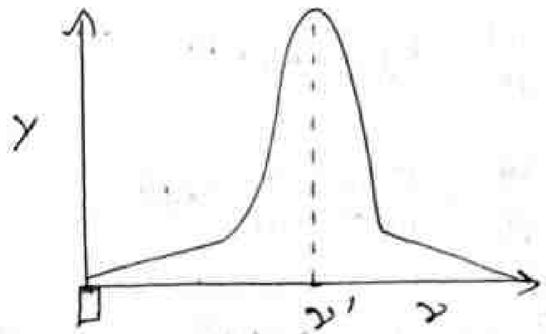
2. Resonance:-

It is a special case of forced vibration.

The phenomenon of making a body vibrate with its natural frequency under the influence of another vibrating body with same frequency is called resonance.



Theory of resonant vibrations:-

Condition of amplitude resonance, In case of forced vibrations,

$$A = \frac{f}{\sqrt{(\omega - p^2)^2 + 4b^2 p^2}}$$

For a particular value of P. the amplitude becomes maximum this phenomenon is known as amplitude resonance.

The amplitude is maximum when,

$$\sqrt{(\omega - p^2)^2 + 4b^2 p^2} \text{ minimum.}$$

$$\frac{d}{dp}\left[(\omega - p^2)^2 + 4b^2 p^2\right] = 0$$

or) $2(\omega - p^2)^2(-2p)^2 + 4b^2(2p) = 0.$

$$\omega^2 - p^2 = 2b^2, \quad p = \sqrt{\omega^2 - 2b^2}.$$

Thus the amplitude is maximum when the frequency $\frac{p}{2\pi}$ of the impressed force becomes $\frac{\sqrt{(\omega^2 - 2b^2)}}{2\pi}$.

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condition for amplitude Resonance:-

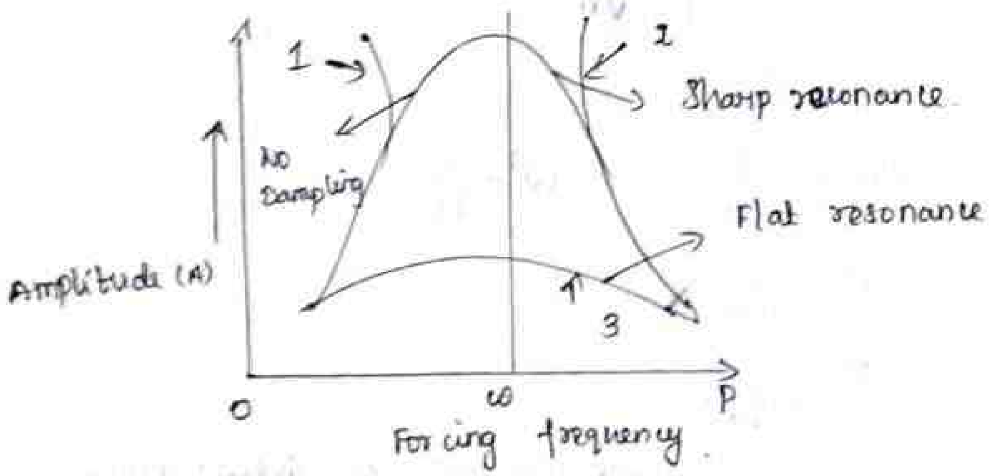
using eqn (1). A is maximum only when,

for negligible damping $b=0$ and,

$$\boxed{A_{max} = \frac{f}{2bp}} \qquad b \to 0,\ A_{max} \to \infty.$$

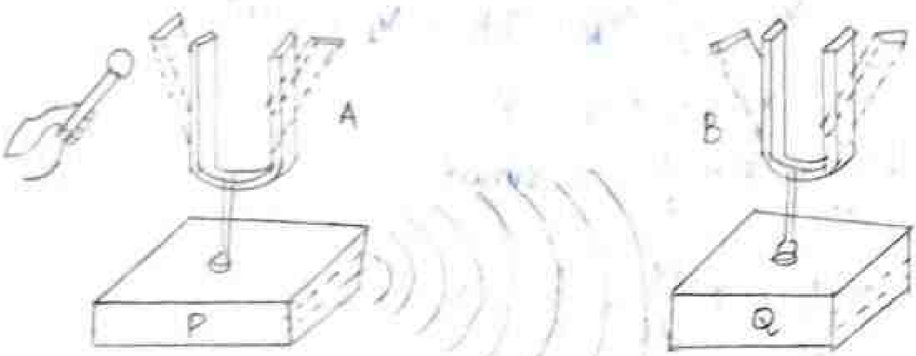
Sharpness of Resonance:-

The rate of change of amplitude with the change of forcing frequency on each side of resonant frequency is known as sharpness of resonance.



Example:-

Two tuning forks of same frequency are mounted on a suitable sound boards and arranged,



If one is made to vibrate by striking it with rubber hammer, it is found that the second fork is also set in vibrations.

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3. Mechanical And Electrical Analogues:-

In a Mechanical vibration, the particles have both kinetic energy and potential energy. The total energy is sum of these two energies.

The equation of motion is given by $\frac{d^2y}{dt^2}+\omega^2 y=0$

In the case of spring.

$$f=\frac{1}{2\pi}\sqrt{\frac{K}{m}}$$

$$\omega=2\pi f=\frac{2\pi}{T}=\sqrt{\frac{K}{m}}$$

$$f=\frac{1}{2\pi}\sqrt{\frac{1}{LC}}$$

$$L\omega=\frac{1}{C\omega}\Rightarrow\omega^2=\frac{1}{LC}$$

$$\omega=\frac{1}{\sqrt{LC}}$$

$$f=\frac{1}{2\pi\sqrt{LC}}$$

It consists of a capacitance (C). an inductance (L) and a resistance (R)

$V=V_0\sin\omega t$ is applied voltage.

$$V_C=\frac{q}{C},\ V_R=IR\ ,\ V_L=L\,dI$$

$$V_L+V_R+V_C=V$$

i.e., $L\frac{dI}{dt}+IR+\frac{q}{C}=V_0\sin\omega t$ $\qquad\because I=\frac{dq}{dt}$

i.e., $L\frac{d^2q}{dt^2}+R\frac{dq}{dt}+\frac{q}{C}=V_0\sin\omega t$

or. $\frac{\cancel{L}}{\cancel{L}}\frac{d^2q}{dt^2}+\frac{R}{L}+\frac{q}{CL}=\frac{V_0}{L}\sin\omega t \qquad \cdots(1)$

The eqn (1) is similar to eqn of motion for a motion forced vibration.

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$$\frac{d^2x}{dt^2} + 2k\frac{dx}{dt} + \omega_0^2 x = f\sin\omega t \quad \ldots\ldots (2)$$

where $2k = \frac{r}{m}$, $\omega_0^2 = \frac{S}{m}$ & $f = \frac{F}{m}$

It is noted that the mass 'm' is analogous to self inductance L, r to the electrical resistance R, compliance $1/S$ the electrical capacitance. C.

Difference between Mechanical quantities and Electrical Analogues

Mechanical Quantities	Electrical Analogues
* Displacement (x)	charge (q)
* ~~mass~~ velocity $\left(\frac{dx}{dt}\right)$	current $(I = dq/dt)$
* mass (m)	Inductance (L)
* Force (F)	voltage (V_0)
* Damping constant (r)	Resistance (R)
* Stiffness constant (s)	Reciprocal of capacitance $\left(\frac{1}{C}\right)$
Quality factor $Q = \frac{\omega' m}{r}$	Quality factor $Q = \frac{\omega' L}{R}$
* For mechanical oscillator	For electrical oscillator
* Damping co-efficient $k = \frac{r}{2m}$	Damping co-efficient $k = \frac{R}{2L}$
* Mechanical Impedance	Electrical Impedance
* $Z = \sqrt{\left(\frac{s}{\omega} - m\omega\right)^2 + r^2}$	$(Z) = \sqrt{\left(\frac{1}{\omega C} - \omega L\right)^2 + R^2}$
* Relaxation time $\left(\frac{1}{k}\right) = \frac{2m}{r}$	Relaxation time $\left(\frac{1}{k}\right) = \frac{2L}{R}$
* Potential energy $= \frac{1}{2}sx^2$	Electrostatic energy $= \frac{q^2}{2C}$

4. Waves on String :-

A string is a cord whose length is very long compared to its diameter and which is uniform and flexible.

Vibrations of stretched string :-

If the string vibrates with nodes at the fixed ends and an antinode at the centre, then it is said to vibrate in fundamental mode.

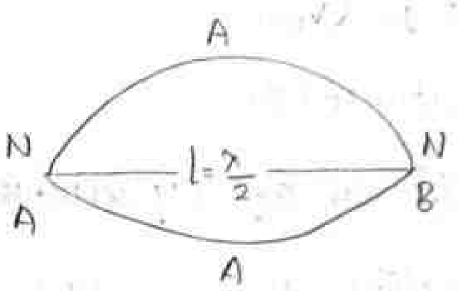
The frequency corresponding to this mode of vibration is known as frequency of fundamental mode.

The speed of the wave is given by.

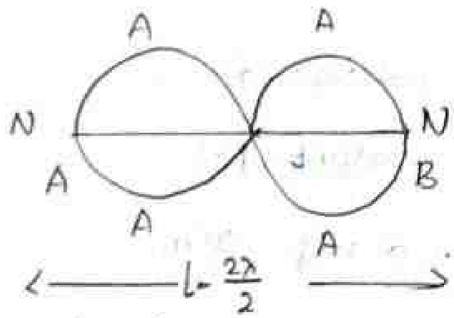
$$\boxed{v = \sqrt{\frac{T}{m}}}$$

$T \rightarrow$ is tension in the string and

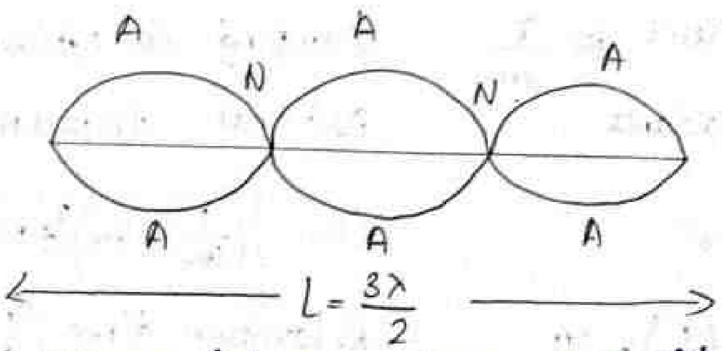
$m \rightarrow$ is mass per unit length of string.



Fundamental or first harmonic



Second harmonic



The distance between two consecutive nodes is equal to $\lambda/2$, where λ is wavelength.

If 'L' be the length between fixed ends of string.

$$L = \frac{\lambda}{2} \text{ or } \lambda = 2L \quad \ldots (2)$$

If 'n' be the frequency of vibration of string, then

$$\boxed{n = \frac{v}{\lambda} = \frac{v}{2L}} \quad \ldots (3)$$

Substituting for v from eqn (1).

$$n = \frac{1}{2L}\sqrt{\frac{T}{m}}$$

T - tension in the string

m - mass per unit length of string

l - length of string

fundament frequency $\boxed{(n) = \frac{P}{2L}\sqrt{\frac{T}{m}}}$

where p is number of loops and it takes values as $p = 1, 2, 3 \ldots$

$P = 1$, $n_1 = \frac{1}{2L}\sqrt{\frac{T}{m}}$ called fundamental frequency.

$P = 2$, $n_2 = \frac{2}{2L}\sqrt{\frac{T}{m}} = 2n$ called 1^{st} over tone.

Laws of Transverse vibrations of stretched strings:-

The frequency of vibration of the fundamental note of a stretched string is given by.

$$\boxed{n = \frac{1}{2L}\sqrt{\frac{T}{m}}}$$

where T is the ~~tension~~ tensi and m is mass per unit length of string.

i) Law of Length:-

The fundamental frequency of vibration of a stretched string is inversely proportional to the length, when the when the tension and the mass per unit length of the string remain constant.

i) $n \propto \frac{1}{L}$ when T and m are constant.

(or) $\boxed{nL = \text{constant}.}$

ii) Law of tension:-

The frequency of vibration of a stretched string is directly proportional to the square root of tension, when the length and the mass per unit length of the string remain constant.

$n \propto \sqrt{T}$ when L and m are constant

(or) $\boxed{\frac{n}{\sqrt{T}} = \text{constant}}$

iii) Law of mass:-

The fundamental frequency of vibration of a stretched string is inversely proportional to the square root of the mass per unit length when the tension and the length of the string remain constant.

i) $n \propto \frac{1}{\sqrt{m}}$ when T and L are constant.

(or) $\boxed{n\sqrt{m} = \text{constant}}$

wave motion:-

* An important type of motion that occurs in nature is wave motion.

* A wave motion is a disturbance of some kind which moves from one place to another by means of a medium such that the medium itself is not transported.

Types of wave motion:-

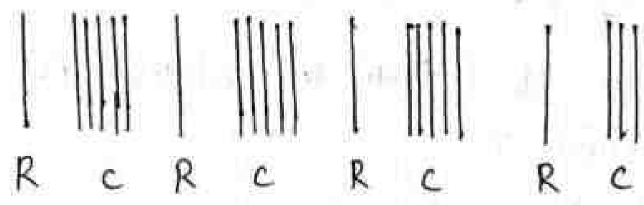
There are two types of wave motion

They are,

1. Longitudinal wave motion.
2. Transverse wave motion.

1. Longitudinal wave motion :-

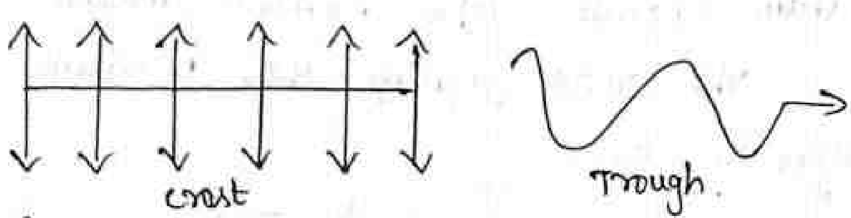
wave motion in which the particles of the medium vibrate about their mean position along the same line as propagation of wave is called Longitudinal wave motion.



eg :- Sound waves.

2. Transverse wave motion :-

wave motion in which particles of the medium vibrate about their mean position at right angle to the direction of propagation is called transverse wave motion.



eg :- waves on the surface of water.

5. Plane progressive waves OR Traveling waves :-

progressive wave originating from a point source and propagating through an isotropic medium travel with equal velocity in all directions.

At any instant, the wavefront will be spherical in nature.

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Relation between frequency, wave speed and wavelength.

By definition the distance travelled by the wave in one time period (T) of vibration of particle = wave length (λ)

i.e. Distance travelled in T second = λ

$\therefore$ Distance travelled in one second = λ/T

but distance travelled in one second = wave speed v.

$$\therefore \frac{\lambda}{T} = v \text{ or } \lambda = vT \quad \ldots (1)$$

Substituting value of T from the relation between frequency (n) and time period T.

i.e. $T = \frac{1}{n}$ in (1), $\lambda = \frac{v}{n}$, $v = n\lambda$

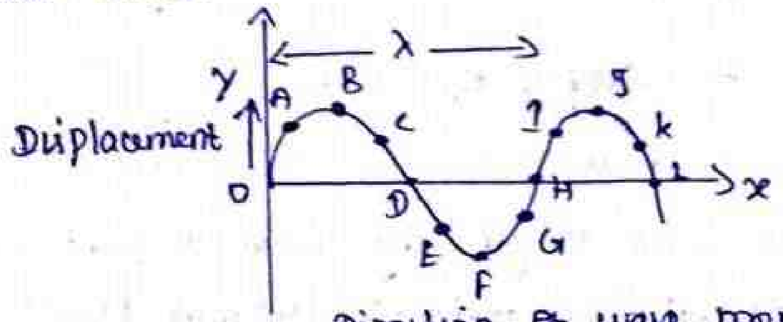
$$\boxed{\text{Speed of the wave } v = \text{frequency } (n) \times \text{wavelength } (\lambda)}$$

This relation holds for all types of waves.

Wave equation of a plane - progressive wave:

On propagation of wave in a medium, the particles of medium execute simple harmonic motion.

The curve joining these positions represents the progressive wave.



Direction of wave propagation.

$$\boxed{y = A\sin\omega t} \quad \ldots (2)$$

A is amplitude and ω is angular velocity.

If n is frequency of wave, then $\omega = 2\pi n$.

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The displacement of particle O at time $\left(t - \frac{x}{v}\right)$ can be obtained by substituting $\left(t - \frac{x}{v}\right)$ in place of t in eqn (1).

Thus the displacement of particle 'C' at a distance x from origin O at any time t is given by,

$$\boxed{y = A \sin \omega \left(t - \frac{x}{v}\right)} \quad \ldots (2)$$

If T is time-period and λ the wavelength of wave,

$$\omega = \frac{2\pi}{T}$$

$$\therefore \quad y = A \sin \frac{2\pi}{T}\left(t - \frac{x}{v}\right)$$

$$= A \sin 2\pi \left(\frac{t}{T} - \frac{x}{vT}\right)$$

But $vT = \lambda$

$$\therefore \quad y = A \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda}\right) \quad \ldots (3)$$

$$y = A \sin \frac{2\pi}{\lambda}\left(\frac{t\lambda}{T} - x\right).$$

$$\therefore \quad y = A \sin \frac{2\pi}{\lambda}(vt - x) \quad \ldots (4)$$

$$\left(\because \lambda/T = v\right)$$

$$y = \sin \left(\frac{2\pi v t}{\lambda} - \frac{2\pi x}{\lambda}\right)$$

$$y = \sin \left(\frac{2\pi n v t}{\lambda} - \frac{2\pi n x}{v}\right) \qquad \left[\because n\lambda = v \quad \frac{n}{v} = \frac{1}{\lambda}\right]$$

The eqn (2) is also expressed as,

$$y = A \sin \left(\omega t - \frac{\omega}{v} x\right) \qquad \left[\because 2\pi n = \omega\right)$$

$\frac{\omega}{v} = \frac{2\pi}{\lambda} = k \Rightarrow$ propagation constant.

$$\boxed{y = A \sin (\omega t - kx)} \quad \ldots (5)$$

substituting $-x$ for x, so that eqn (5).

$$\boxed{y = A \sin (\omega t + kx)}$$

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If ϕ is the phase difference between this wave travelling along positive x-axis and another wave,

$$\boxed{y = A\sin[(\omega t - kx) + \phi]} \quad \ldots (7)$$

Differential eqn of wave motion :-

$$y = A\sin \frac{2\pi}{\lambda}(\nu t - x)$$

$$\frac{dy}{dt} = \frac{2\pi \nu A}{\lambda} \cos \frac{2\pi}{\lambda}(\nu t - x) \quad \ldots (8)$$

$$dy/dx = \frac{-2\pi A}{\lambda} \cos \frac{2\pi}{\lambda}(\nu t - x) \quad \ldots (9)$$

∴ particle velocity.

$$\frac{dy}{dt} = -\nu \frac{dy}{dx} \quad \ldots (10)$$

from eqn (9).

$$\frac{d^2y}{dx^2} = -A\left(\frac{2\pi}{\lambda}\right)^2 \sin \frac{2\pi}{\lambda}(\nu t - x) \quad \ldots (11)$$

From eqn (8)

$$\frac{d^2y}{dt^2} = -A\left(\frac{2\pi}{\lambda}\right)^2 \nu^2 \sin\frac{2\pi}{\lambda}(\nu t - x) \quad \ldots (12)$$

This particle acceleration.

Comparing (11) & (12)

$$\frac{d^2y}{dt^2} = \nu^2 \frac{d^2y}{dx^2} \quad \ldots (13)$$

(or)

$$\frac{d^2y}{dx^2} = \frac{1}{\nu^2}\frac{d^2y}{dt^2}$$

It can be shown that in case of progressive waves, if t is increased by δt and x by $\nu \delta t$..

$$y' = a\sin\frac{2\pi}{\lambda}[\nu(t + \delta t) - (x + \nu\delta t)]$$

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* The velocity of the particles at the nodes is zero. It increases gradually and is maximum at the antinodes.
* Distance between any two consecutive nodes or antinodes is equal to $\lambda/2$, whereas the distance between a node and its adjacent antinode is equal to $\lambda/4$.

Difference between progressive waves and stationary waves:-

S. No.	progressive waves	stationary waves
1.	There is transfer of energy in the direction of propagation of wave	There is no transfer of energy
2.	No particle of the wave is permanently at rest	The particles at nodes are permanently at rest
3.	The particles of the medium vibrate with same amplitude about their mean.	Amplitude of each particle is not same. It is maximum at antinodes and decreases gradually to zero at the nodes.
4.	The phase of vibration varies continuously	Particles in the same segment vibrate in the same phase

7. Energy Transfer of a wave:-

The mechanical energy is transferred through the vibration of the string.

Let us consider a string under uniform tension T and m the mass / unit length.

If a small element of the string of length dx is considered then its mass is m dx.

$$dk = \frac{1}{2} m \,(\text{velocity})^2$$

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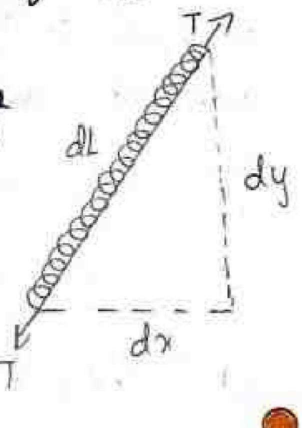
$$dk = \frac{1}{2} m.dx \left(\frac{dy}{dt}\right)^2 \quad \ldots (1)$$

where, dy is the vertical displacement of the portion of a string. x is the direction of propagation of the wave in a string.



$$dl = \sqrt{(dx)^2 + (dy)^2} = dx\sqrt{1 + \left(dy/dx\right)^2}$$

$$= dx\left(1 + \frac{1}{2}\left(\frac{dy}{dx}\right)^2\right) \ldots (2)$$

$$\Delta l = dl - dx = \frac{1}{2}\left(\frac{dy}{dx}\right)^2 dx \quad \ldots (3)$$

If the string is vibrating with displacement.

$$y(x,t) = A\cos(\omega t - kx) \quad \ldots (4)$$

therefore, the potential energy is,

$$dU = T.\Delta l = \frac{1}{2} T \left(dy/dx\right)^2 . dx \quad \ldots (5)$$

Substituting (4) in (5) gives.

$$dU = \frac{1}{2} Tk^2 A^2 \sin^2(\omega t - kx).dx$$

$$dU = \frac{1}{2} mA^2 \omega^2 \sin^2(\omega t - kx).dx \quad \ldots (6)$$

where $v = \frac{\omega}{k}$, $v = \sqrt{\frac{T}{m}}$, $T = mv^2$

$$Tk^2 = T.\frac{\omega^2}{v^2} = T.\frac{\omega^2}{(T/m)} = m\omega^2$$

If we substitute eq (4) in (1)

$$dk = \frac{1}{2} mA^2\omega^2 \sin^2(\omega t - kx).dx \quad \ldots (7)$$

comparing eqn (4) and eqn (6)

$$dU = dk \quad \ldots (8)$$

$\therefore$ The total energy is,

$$dE = dU + dk = 2dk = mA^2\omega^2 \sin^2(\omega t - kx) \quad \ldots (9)$$

(or) $dE = m(A\omega)^2 \sin^2(\omega t - kx)\,dx \quad \ldots (10)$

The following points may be noted about sound waves.
Sound waves are longitudinal waves
Material medium is necessary for the transmission of sound from one place to another.
The velocity of sound is greater in solids and liquids than in gases.

Audiable and Inaudiable sounds:-

* Those sounds which human ear can hear are called audible sounds. The range of human hearing is 20 Hz to 20 kHz. In other words, we cannot hear sounds of frequency below 20 Hz or above 20 kHz.
* Those sounds which human ear cannot hear are called inaudible sounds. The sounds of frequency below 20 Hz are called infrasonics while sounds of frequency above 20 Hz are called ultrasonics.
* If infrasonics and ultrasonics find wide applications in science and engineering.

Velocity of sound in a medium:-

The velocity of sound in a medium depends upon elasticity and density of the medium. According to Newton's formula, the velocity v of sound is given by.

$$v = \sqrt{\frac{E}{\rho}}$$

where, E = modulus of elasticity of the medium
ρ = density of the medium.

i) For solids $E = Y$ (young's modulus of elasticity)

$$\therefore \boxed{v = \sqrt{Y/\rho}}$$

The quantity $\frac{dE}{dx}$ is called the linear energy density.

$$\left(\frac{dE}{dx}\right)_{Average} = d\vec{E} = \frac{1}{2} m (A\omega)^2 \quad \dots (11)$$

The relation is called as average energy density $d\vec{E}$

As the average power transmitted by the wave is,

$$\vec{P} = \left(\frac{dE}{dt}\right)_{Average}$$

eq (11) becomes,

$$(dE)_{Average} = \frac{1}{2} m A^2 \omega^2 . dx$$

$$\vec{P} = \frac{1}{2} m A^2 \omega^2 \frac{dx}{dt}$$

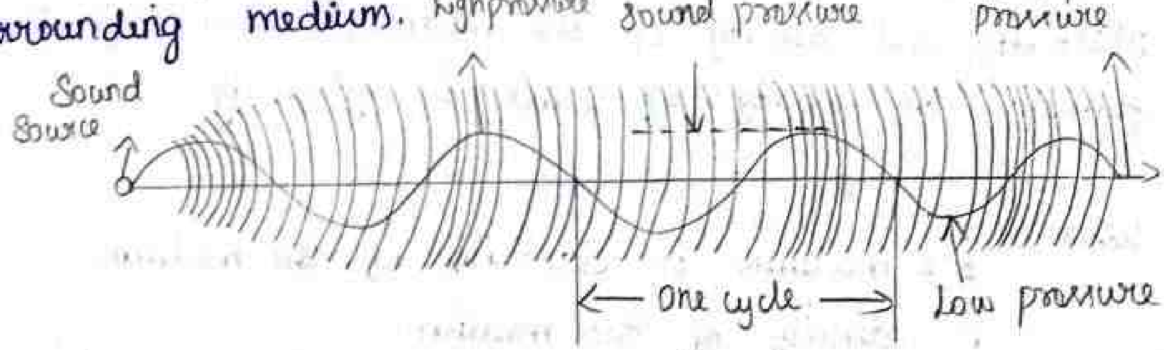
$$\vec{P} = \frac{1}{2} m A^2 \omega^2 . v$$

where $v = \frac{dx}{dt}$ is the wave velocity.

eq (13) states that wave power is directly proportional to the speed or velocity v at which energy moves along the wave.

8 Sound waves:-

Sound is always produced by some vibrating body. The vibrating body excites mechanical waves in the surrounding medium.



The propagation of sound requires the presence of an elastic medium. Sound cannot travel in a vacuum.

ii) For liquids, $E = K$ (Bulk modulus of elasticity)

$$\therefore \nu = \sqrt{\frac{K}{\rho}} \quad \ldots (2)$$

iii) For gases, $E = K$ (Bulk modulus of elasticity)

$$\nu = \sqrt{\frac{K}{\rho}} \quad \ldots (3)$$

The pressure of the gas.

$$\nu = \sqrt{\frac{P}{\rho}}$$

Velocity of sound in air by Newton's formula:-

According to Newtons formula, the velocity ν of sound in air is given by.

$$\nu = \sqrt{\frac{P}{\rho}}$$

$P = 0.76 \times 13,600 \times 9.8 \, N/m^2$; $\rho = 1.298 \, kg/m^3$

Velocity of sound in air at N.T.P.

$$\nu = \sqrt{\frac{0.76 \times 13.600 \times 9.8}{1.293}} = 280 \, m/s.$$

The speed of sound in air is commonly taken as 344 m/s for normal conditions. This is very less compared to the velocity of light.

---

9. Doppler Effect:-

This apparent change in frequency was first observed by Doppler in 1845.

Definition:-

The phenomenon of the apparent change in the frequency of the sound due to relative motion between the source of sound and the observer is called Doppler effect.

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1. Both source and observer at rest:-

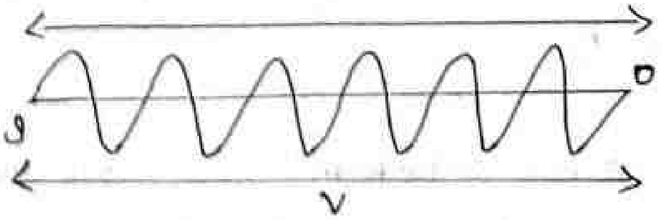
* Suppose S and O are the positions of the source and the observer respectively.

* Let n be the frequency of the sound and v be the velocity of sound.

* In one second n waves produced by the source travel a distance $SO = v$.

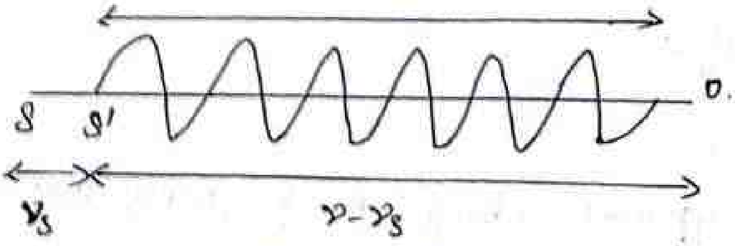
The original wavelength is $\lambda = \frac{v}{n}$ ... (1)

The original frequency $n = \frac{v}{\lambda}$ ... (2).



2. When the source moves towards the stationary observer:-

If the source moves with a velocity v_s towards the stationary observer then after one second the source will reach S' such that $SS' = v_s$.



The apparent wavelength of the sound,

$$\lambda' = \frac{v - v_s}{n} \quad \ldots (3)$$

The apparent frequency,

$$n' = \frac{v}{\lambda'} = \left(\frac{v}{v - v_s}\right) n \quad \ldots (4).$$

Comparing eqn (2) & (4) we can conclude that $n' > n$, the pitch or frequency of the sound appears to increase.

3. when the sources moves away from the stationary observer:-

If the source moves away from the stationary observer with velocity v_s.

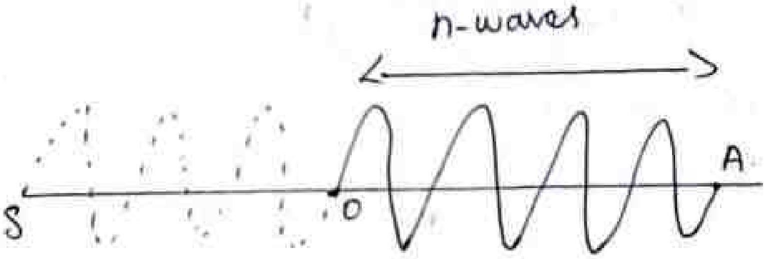
The apparent frequency.

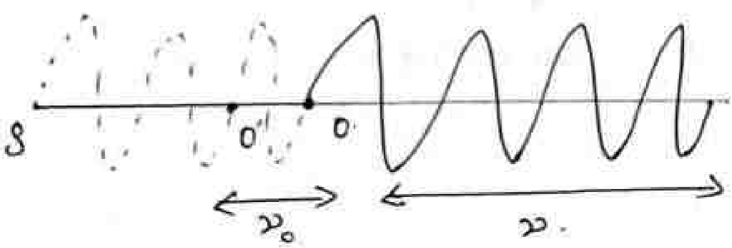
$$\boxed{n' = \frac{v}{\lambda'} = \left(\frac{v}{v-(-v)}\right)n = \left(\frac{v}{v+v_s}\right)n} \quad \cdots (5)$$

Comparing eqn (2) & (5) we can conclude that $n' < n$, the pitch or frequency of the sound appears to decrease.

4. source is at rest and observer in motion:-

S and O represent the position of source and observer respectively. The source S emits n waves per second having a wavelength $\lambda = \frac{v}{n}$





5. when the observer moves towards the stationary source:-

Suppose the observer is moving towards the stationary source with velocity v_0. After one second the observer will reach the point O' such that $OO' = v_0$

Therefore, the apparent frequency of sound. 21

$$n' = n + \frac{v_0}{\lambda} = n + \left(\frac{v_0}{v}\right) n$$

The apparent frequency of sound.

$$n' = \left(\frac{v - v_0}{v}\right) n \quad \ldots (6)$$

Comparing equations (2) and (6) we can conclude that $n' > n$ the pitch of the sound appears to increase.

Q. 6. When the observer moves is away from the stationary source.

Suppose the observer is moving towards the stationary source with velocity v_0.

Therefore the apparent frequency of sound.

$$n' = \left(\frac{v + (-v_0)}{v}\right) n$$

The apparent frequency of sound.

$$\boxed{n' = \left(\frac{v - v_0}{v}\right) n} \quad \ldots (7)$$

Comparing eqn (2) and (7) we can conclude that $n' < n$ the pitch of the sound appears to decrease.

Applications of Doppler effect:-

(i) To measure the speed of an automobile:-

An electromagnetic wave is emitted by a source attached to a police car. The wave is reflected by a moving vehicle, which acts as a moving source.

ii) RADAR (Radio detection and ranging).

A RADAR sends high frequency radiowaves towards an aeroplane.

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The difference in frequency is used to determine the speed of an aeroplane.

iii) SONAR (Sound navigation and ranging).

Sound waves generated from a ship fitted with SONAR are transmitted in water towards an approaching submarine.

iv) Blood flow meter:-

Ultrasonic sounds are transmitted towards organs the frequency change in reflected waves used to measure blood flow rate.

v) Tracking a Satellite:-

The frequency of radio waves emitted by a satellite decreases as the satellite passes away from the earth.

vi) Stars moving towards the earth or away from the earth.

* There is an apparent change in wavelength of spectral lines emitted by a moving star.
* The spectral shift enables the velocity of star to be computed along the line of sight.

Optics:-

1. Reflection of light waves:-

* The phenomenon where the incident light falling from one medium on a surface of another medium is sent back to the same medium is known as reflection.
* The angle between the incident ray and the normal to the surface is known as angle of incidence.
* The angle between the reflected ray and the normal of the surface is known as angle of reflection.

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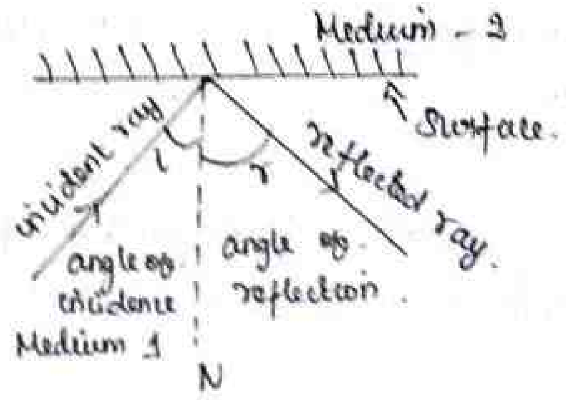


Laws of reflection:

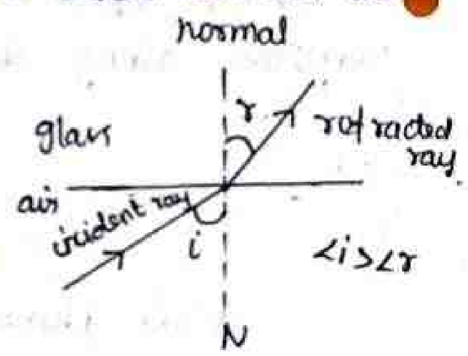
(i) Incident ray, normal and reflected ray lie in the Same plane.

ii) The angle of incidence is equal to the angle of reflection. i.e $\angle i = \angle r$.

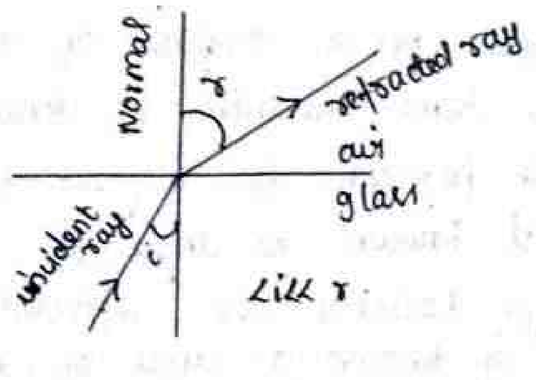
2. Refraction of Light waves:

* Refraction is the phenomenon in which light travels from one medium to another medium. the direction of light changes due to change in medium.

* If a ray of light passes from a rarer medium into denser medium then the ray of light bends towards the normal.



* If a ray of light passes from denser medium into rarer medium then the ray of light bends away from the normal.



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Law of refraction :-

i) The incident ray, the refracted ray and the normal at a point of separation of two media lie in the same plane.

ii) for any two medium, the ratio of sine of angle incidence to sine of angle of refraction is constant. It is known as Snell's law. $\frac{\sin i}{\sin r}$ = constant.

where this constant is called as refractive index (μ) of medium.

$$\therefore \boxed{\mu = \frac{\sin i}{\sin r}}$$

Significance of refractive index :-

The ratio of velocity of light in vaccum to velocity of light in medium is called as refractive index.

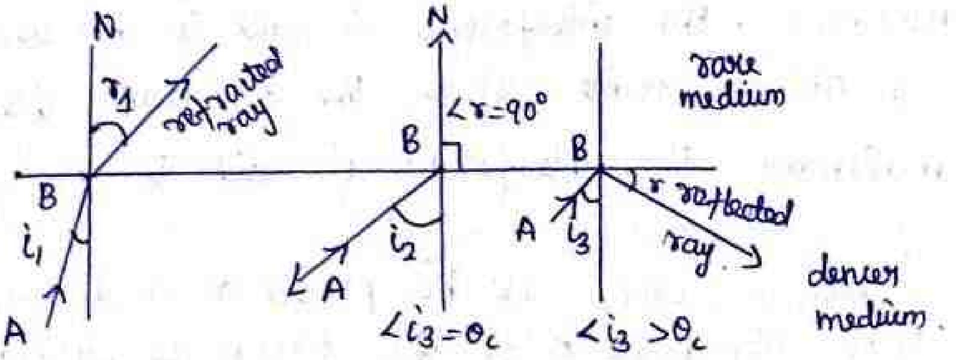
$$\boxed{\text{Refractive index } \mu = \frac{\text{velocity of light in vaccum } (c)}{\text{velocity of light in medium } (v)}}$$

$$\mu = \frac{c}{v}$$

Refractive index for vacuum is unity (1).

3. Total internal Reflection :-

* when light passes from denser medium to rarer medium then the refracted ray bends away from the normal.
* consider a ray AB incident at $\angle i_1$ and refracted at $\angle r_1$, as angle of incidence increases, angle of refraction also increases.



Thus a ray travelling from denser medium to a rarer medium is reflected into denser medium if angle of incidence is more than the critical angle of medium.

Critical angle:-

Definition:-

* The angle of incident at which the refracted ray just graze surface between denser and rarer media is called critical angle.

* When light travels from denser to rarer medium, from Snell's Law,

$$n_1 \sin i = n_2 \sin r$$

n_1 - refractive index of denser medium

n_2 - refrative index of rarer medium

i - angle of incident and r - angle refraction

$i = \theta_c$ when $r = 90°$

$$\therefore n_1 \sin \theta_c = n_2 \sin 90° \qquad (\because \sin 90° = 1)$$

$$\sin \theta_c = \frac{n_2}{n_1}$$

$$\boxed{\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)}$$

4. Interference of Light waves:-

* The modification or change of intensity of light resulting from the superposition of two or more waves of light is called interference.

* At the points, where the resultant intensity of light is maximum, the interference is said to be constructive.

* At the points, where the resultant intensity of light is minimum, the interference is said to be destructive.

Theory of interference fringes:-

Let a screen xy be placed at a distance D parallel to $S_1 S_2$. The point 'O' on the screen is equidistance from S_1

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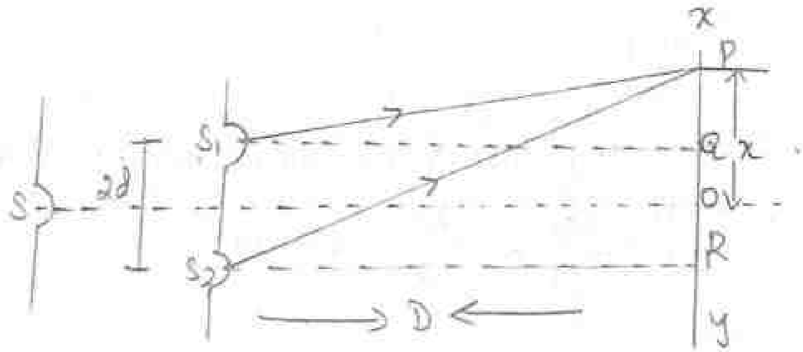
and S_2, consider a point p at a distance x from O.



From the right angled triangle S_1QP

$$(S_1P)^2 = (S_1Q)^2 + (QP)^2$$

(or) $(S_1P)^2 = D^2 + (x-d)^2$ ... (1) $[\because QP = (x-d)]$

Similarly in right angle triangle S_2RP

$$(S_2P)^2 = (S_2R)^2 + (RP)^2$$

(or) $(S_2P)^2 = D^2 + (x+d)^2$ ... (2) $[\because RP = (x+d)]$

$\therefore (S_2P)^2 - (S_1P)^2 - (S_1P)^2 = D^2 + x^2 + d^2 + 2dx - D^2 - x^2 + 2dx - d^2$

$$(S_2P)^2 - (S_1P)^2 = 4xd$$

(or)

$$(S_2P - S_1P)(S_2P + S_1P) = 4xd$$

the error is not more than a fraction of one percent.

$$(S_2P - S_1P)2D = 4xd$$

(or) $(S_2P - S_1P) = \frac{4xd}{2D} = \frac{2xd}{D}$ ... (1)

Position and spacing of fringes:-

Now, we shall consider the following two cases:

Bright fringes:- The point p is bright when the path difference is a whole number multiple of wavelength λ.

$$S_2P - S_1P = n\lambda \qquad n = 0, 1, 2.$$

Substituting the value of from eqn (1).

$$\frac{2xd}{D} = n\lambda \quad \text{(or)} \quad x = \frac{n\lambda D}{2d} \quad \text{... (2)}$$

when $n = 1, 2, 3 \ldots$, $n = 1, x_1 = \frac{\lambda D}{2d}$

$n = 2, x_2 = \frac{\lambda D}{2d}$

The distance between any two consecutive bright fringes.

$$x_2 - x_1 = \frac{2\lambda D}{2d} - \frac{\lambda D}{2d} = \frac{D\lambda}{2d} \quad \ldots (3)$$

Dark fringes:-

The point p is dark. when the path difference is an odd number multiple of half wavelength.

$$(S_2P - S_1P) = (2n+1)\frac{\lambda}{2} \qquad n = 0, 1, 2, 3.$$

$$\frac{2xd}{D} = \frac{(2n+1)\lambda}{2}$$

$$x = \frac{(2n+1)\lambda D}{4d} \quad \ldots (4)$$

When,

$n = 0, \; x_0 = \frac{\lambda D}{4d}$

$n = 1, \; x_1 = \frac{3\lambda D}{4d}$

$n = 2, \; x_3 = \frac{5\lambda D}{4d}$

The distance between any two consecutive dark fringes.

$$x_2 - x_1 = \frac{5\lambda D}{4d} - \frac{3\lambda D}{4d} = \frac{2\lambda D}{2 \cdot 4d} = \frac{\lambda D}{2d}$$

This is expressed by β $\left(\beta = \frac{\lambda D}{2d}\right)$ and it is known as fringe width.

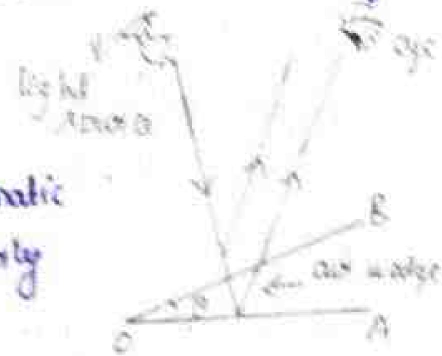
---

5. Theory of air wedge and experiment:-

Air-wedge arrangement is used to find the thickness of a thin sheet or a wire. It is also used to test the planeness of the glass plate.

Definition:

A wedge shaped air film enclosed in between of two glass plates is called air wedge.

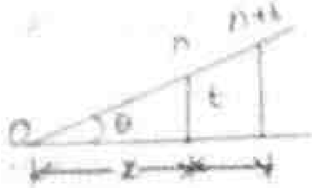


Theory of air wedge experiment:

The light rays from a monochromatic light source is made to fall perpendicularly on the film.

These two reflected rays will interfere and a large number of straight alternative bright and dark fringes are formed.

If t is the thickness of the air film corresponding to the n^{th} dark band with wedge angle θ at a distance of x metre from the edge of contact.



for air film, refractive index of the film $\mu = 1$. $\quad 2\mu t \cos r = n\lambda \quad \ldots (1)$

$\cos r = 1,\ r = 0,\ \cos 0 = 1.$

where λ - wavelength light

$$2t = n\lambda \quad \ldots (2)$$

$$\frac{t}{x} = \tan\theta \Rightarrow \frac{t}{x} = \theta \qquad (\because \theta \text{ is very small } \tan\theta = \theta)$$

$$\therefore t = x\theta \quad \ldots (3)$$

Substituting eqn (3) in eqn (2). for n^{th} dark band.

$$2x\theta = n\lambda \quad \ldots (4)$$

$$2(x+\beta)\theta = (n+1)\lambda \quad \ldots (5)$$

where β is the fringe width.

Subtracting eqn (4) from eqn (5)

$$2\beta\theta = \lambda$$

$$\boxed{\beta = \frac{\lambda}{2\theta}} \quad \ldots (6)$$

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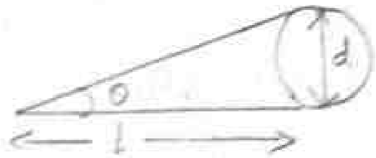
Thickness of a thin wire and very thin foil:-

The given wire whose thickness d is to be measured is placed in between the two glass plates to form a wedge shaped air film.

$$\tan\theta = \frac{d}{l} \quad (\because \tan\theta = \theta)$$

$$\text{or} \quad \theta = \frac{d}{l} \quad \ldots (7)$$



Substituting eqn (7) in (6).

$$\beta = \frac{\lambda}{\frac{2d}{l}} = \frac{\lambda l}{2d}.$$

$$\boxed{d = \frac{\lambda l}{2\beta}}$$

Thus, thickness of very thin specimen can be determined by using the interferance technique in wedge shaped film.

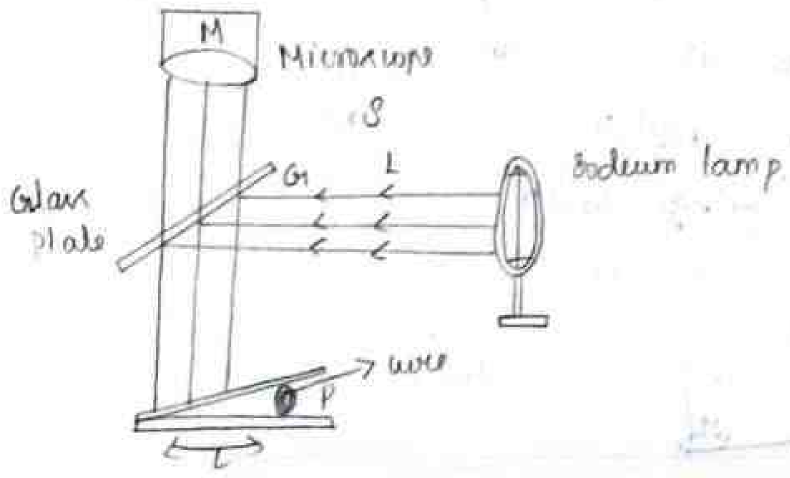
Applications of air-wedge:-

Determination of diameter of a wire or thickness of a thin sheet of paper.

Therefore, glass plates are inclined at a very small angle θ. This is called airwedge arrangement.

Description:-

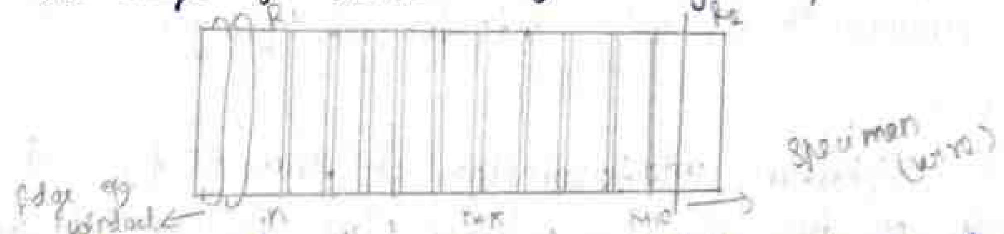
This arrangement is kept on the bed of the travelling microscope.



A parallel beam of monochromatic light from a light source is reflected down on the air wedge by a glass plate kept inclined at an angle 45° to the horizontal.

Experiment:-

Microscope is focussed on these fringes and the vertical cross wire is made to coincide with n^{th} bright near the edge of contact of the glass plates.



The reading on the horizontal scale of the microscope is noted. The cross wire is made to coincide with successive 5^{th} fringes ($n+5$, $+10$ ... $n+40$) and the corresponding microscope readings are noted.

Knowing the wavelength of the monochromatic light source the thickness of the wire is found out using the formula.

$$d = \frac{l\lambda}{2\beta} \text{ metre.}$$

Table:-

S. No.	order of the fringes.	Microscope Reading $\times 10^{-2}$ m	width of 10 fringes. m	Band width β m
1.	n			
2.	$n+5$			
3.	$n+10$			
	⋮			
	$n+40$			

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6. Michelson's interferometer:-

principle:-

Two interfering beams are formed by splitting the light from a source into two parts by partial reflection and refraction.

: These beams are sent in two perpendicular directions and they are finally brought together after reflection from plane mirrors to produce interference fringes.

construction:-

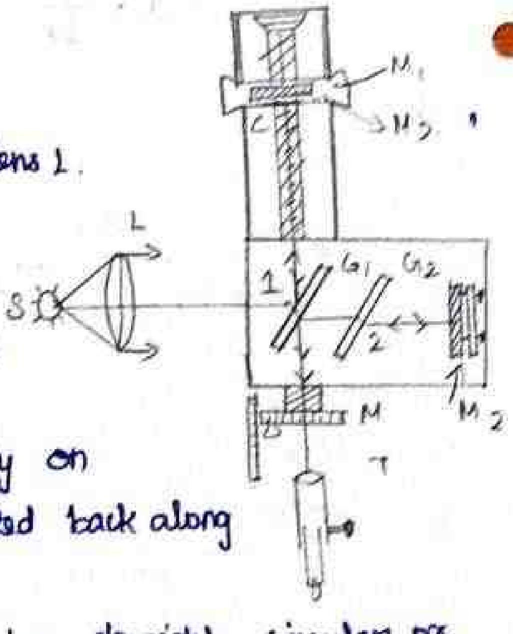
* Michelson interferometer is show in fig. The apparatus consists of two highly polished plane mirrors M_1 and M_2.

* The mirrors are mounted vertically on two arms perpendicular to each other.

* There are two plane parallel glass plates G_1 and G_2 of same thickness placed at an angle of 45^0 to the incident beam.

* A telescope (T) is positioned perpendicular to M_1 to receive rays reflected from both M_1 and M_2.

working:-



* Light from source S is rendered parallel by means of a collimating lens L.

* This light is made to fall on a semi-silvered glass plate G_1.

* The light beam is divided into two parts.

* These light rays fall normally on mirrors M_1 and M_2 and are reflected back along its original paths.

The interference fringes may be straight, circular or parabolic etc. depending upon,

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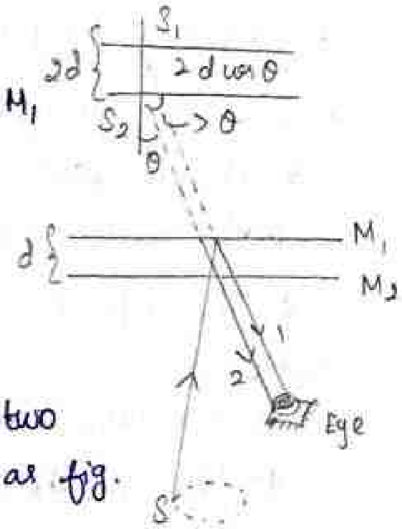
* path difference and
* angle between mirror M_1 and virtual mirror M_2'

Formation of fringes :-



* One of the interfering beams comes from M_1 and the other appears by reflection from the virtual image of mirror M_2. M_2'
* An air film is enclosed between the two mirrors M_1 and M.
* The two interfering beams appear from two virtual images S_1 and S_2 of the light source S as fig.

For maximum intensity in the fringes,

$$2d\cos\theta + \lambda/2 = n\lambda \qquad n = 0, 1, 2 \ldots$$

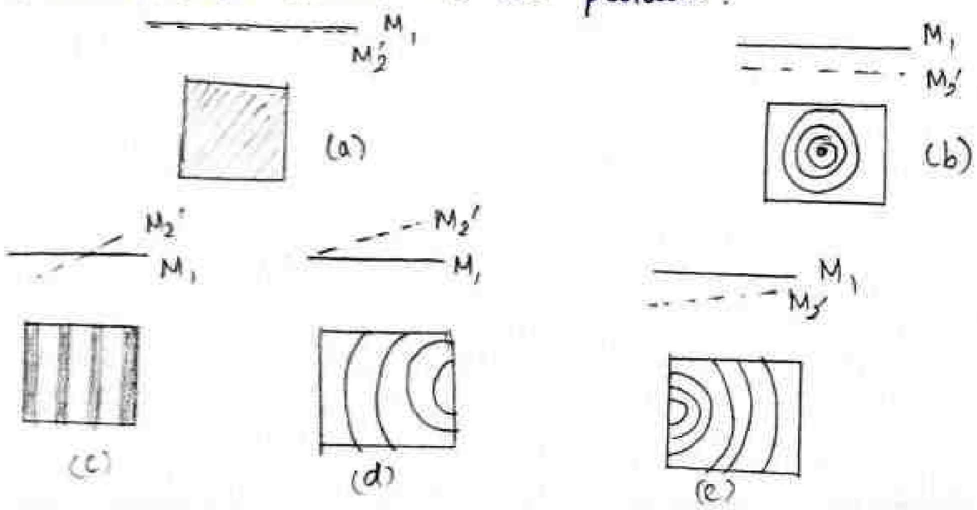
Types of fringes :-

Case (1) :-

* When M_2' coincides with M_1 i.e. the paths are exactly equal, the paths difference is only $\lambda/2$.
* Therefore, the field of view is perfectly dark as shown in fig.

Case (2) :-

M_1 is moved either forward or backward parallel to itself. Now, mirror M_1 is exactly perpendicular to mirror M_2. i.e. Mirror M_1 and virtual mirror M_2' are parallel.



Case 3 :-

when mirror M_1 intersects the virtual image M'_2. the air film enclosed is wedge shaped, and straight line fringes are produced as fig. (c).

Applications of Michelson's Interferometer :

* It is used to find.
* The wavelength of a given light source,
* The refractive index and thickness of a transparent material.
* The resolution of wavelengths.
* the standardisation of meter.

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3. Laser.

①

1. Theory of Laser:-

Interaction of light radiation with materials:-

Consider an assembly of atoms in a material which exposed to light radiation.

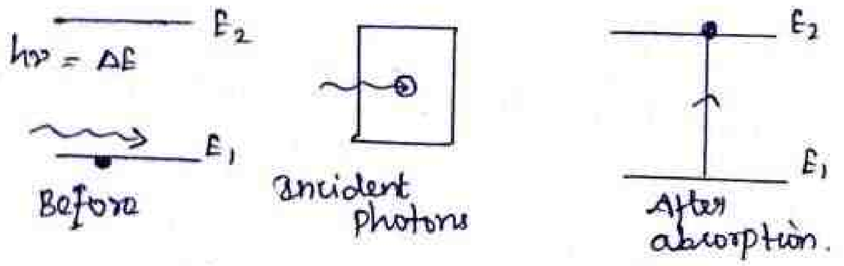
In general, three different processes when light radiation interacts with a material. They are

* Stimulated absorption.
* Spontaneous emission
* Stimulated emission

Stimulated absorption:- (process - 1).

An atom in ground state with energy E_1 absorbs an incident photon of energy $h\nu$ and is excited to higher energy state with energy E_2.

This process is known as stimulated or induced absorption.



For each transition, a certain amount of energy $(h\nu)$ is absorbed from the incident light beam.

* The excited atoms do not stay in the higher energy state for a longer time.

* It is the tendency of atoms in excited state to come to the lower energy state.

②

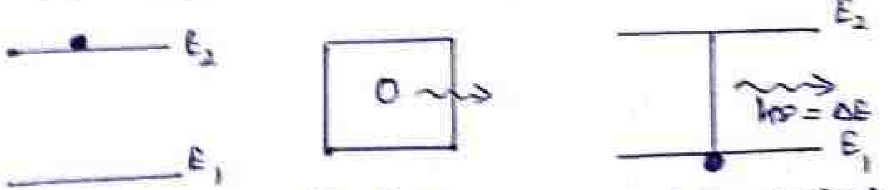
The emission of photons takes place in two ways

* spontaneous emission.
* stimulated emission.

2. <u>spontaneous and stimulated Emissions</u> :-

<u>spontaneous emission</u> :- (process-2)

The atom in the excited state E_2 returns to ground state E_1 by emitting a photon of energy $h\nu$ without the influence of any external agency.



Before emission   Random emission   After emission

Such emission of light radiation which is not triggered by any external influence is called spontaneous emission.

It is a random and also uncontrollable process.

<u>process-3 stimulated emission</u> :

* He found that there is an interaction b/w the atom in excited state and a photon.
* During this interaction, the photon triggers the excited atom to make transition to ground state E_1.
* Such kind of fore-forced emission of photons by the incident photons is called stimulated emission.
* It is also known as induced emission. It plays a key factor for the working of a laser.

2. Einstein's A and B coefficients (Derivation) (3)

* consider an assembly of atoms with different energy states at an absolute temperature T.

* when light radiation is incident on these atoms, three different processes take place. They are.

(a) stimulated absorption

(b) spontaneous emission

(c) stimulated emission

Stimulated absorption

* The atom in the lower energy state E_1 absorbs radiation and is excited to the higher energy level E_2.

* This process is called stimulated or induced absorption.

* the rate of stimulated absorption is directly proportional to number of atoms (N_1) in energy state E_1 and density (Q) of incident radiation.

$$N_{ab} \propto N_1 Q$$

* Therefore, the number of stimulated absorption transitions occurring per unit time is given by.

$$N_{ab} = B_{12} N_1 Q \quad \ldots (1).$$

where B_{12} is a proportionality constant. This process is an upward transition.

* The atoms in excited state return to lower energy state E_1 by emitting a photon of energy $h\nu$ in two ways, (a) Spontaneous emission

(b) Stimulated emission.

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Spontaneous emission: ④

* The atoms in the excited state E_2 return to lower energy state E_1 by emitting a photon of energy $h\nu$ without the influence of any external agency.

* This emission of light radiation is known spontaneous emission.

* The rate of spontaneous emission is directly proportional to the number of atoms in the excited energy state (N_2).

$$N_{sp} \propto N_2$$

Hence, the number of transitions per second is given by.

$$N_{sp} = A_{21} N_2 \quad \ldots (2)$$

A_{21} is a proportionality constant.

This process is a downward transition.

Stimulated emission:

* The rate of transition is directly proportional to the number of atoms in the upper energy level (N_2) and the energy density of incident radiation (Q)

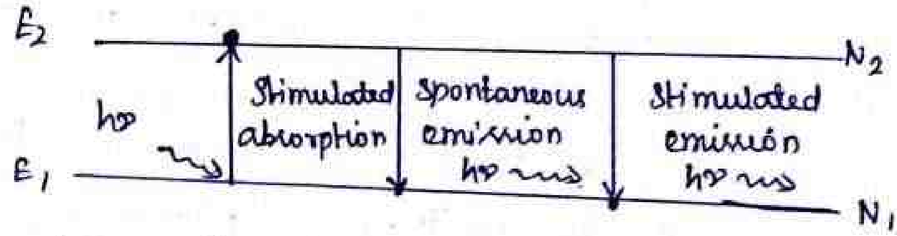
$$N_{st} \propto N_2 Q$$

The number of transitions per second.

$$N_{st} = B_{21} N_2 Q \quad \ldots (3).$$

where B_{21} is a proportionality constant.

This process is also a downward transition.



The proportionality constants A_{21}, B_{12} and B_{21} are known as Einstein's coefficients A and B.

(5)

Under equilibrium condition, the number of downward and upward transitions per second are equal.

$$N_{sp} + N_{st} = N_{ab} \quad \ldots (4).$$

Substituting from the eqn (1), (2) & (3) in eqn (4).

$$A_{21} N_2 + B_{21} N_2 Q = B_{12} N_1 Q \quad \ldots (5)$$

Rearranging the eqn (5).

$$B_{12} N_1 Q - B_{21} N_2 Q = A_{21} N_2$$

$$Q (B_{12} N_1 - B_{21} N_2) = A_{21} N_2$$

$$Q = \frac{A_{21} N_2}{B_{12} N_1 - B_{21} N_2} \quad \ldots (6).$$

Dividing numerator and denominator by $B_{21} N_2$.

$$Q = \frac{\dfrac{A_{21} N_2}{B_{21} N_2}}{\dfrac{B_{12} N_1}{B_{21} N_2} - \dfrac{B_{21} N_2}{B_{21} N_2}}$$

$$Q = \frac{A_{21}}{B_{21}} \frac{1}{\left(\dfrac{B_{12}}{B_{21}}\right)\dfrac{N_1}{N_2} - 1} \quad \ldots (7).$$

on substituting $\frac{N_1}{N_2} = e^{h\nu/kT}$ in eqn (7).

$$Q = \frac{A_{21}}{B_{21}} \frac{1}{\left(\dfrac{B_{12}}{B_{21}}\right) e^{h\nu/kT} - 1} \quad \ldots (8).$$

planck's radiation formula for energy distribution is given by.

$$Q = \frac{8\pi h\nu^3}{c^3} \frac{1}{e^{h\nu/kT} - 1} \quad \ldots (9).$$

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comparing the eqn (8) and (9). ⑥

$$\frac{B_{12}}{B_{21}} = 1 \implies B_{12} = B_{21} \quad \ldots (10).$$

$$\frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3}{c^3} \quad \text{or} \quad = \frac{8\pi h}{\lambda^3} \quad \ldots (11)$$

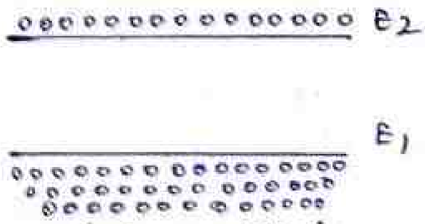
Since $B_{12} = B_{21}$, Einstein's coefficients are termed as A and B coefficients.

Conclusion:-

* The spontaneous emission is more predominant than the stimulated emission.
* The eqn (11) gives the relation b/n spontaneous emission and stimulated emission coefficients.

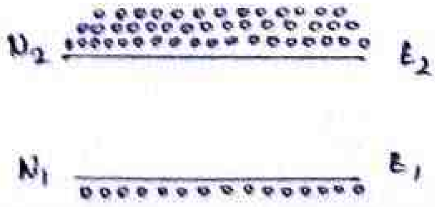
4. Population Inversion:-

It is a situation in which the number of atoms in higher energy state is more than that in lower energy state.



Normal condition

The state of achieving more number of atoms in higher energy state than that of lower energy state is known as population inversion.



conditions for population inversion:- (P)

* There must be atleast two energy levels (E_1 & E_2)
* there must be a source to supply the energy to the medium.
* The atoms must be continuously raised to the excited state.

Active Medium or Material:

* A medium in which population inversion above threshold inversion density is achieved is known as active medium. It is also called active material.
* The inversion density which is just enough to compensate for the losses in the medium is called threshold inversion density.

Pumping action:-

* The process to achieve population inversion in the medium is called pumping action.
* It is an essential requirement for producing a laser beam.

Methods for Pumping action:

The methods commonly used for pumping action are.

* optical pumping
* Electrical discharge
* Direct conversion
* Inelastic collision b/w atoms.

(i) optical pumping:

When the atoms are exposed to light radiation of energy $h\nu$, atoms in the lower energy state absorb these radiation and go to an excited state

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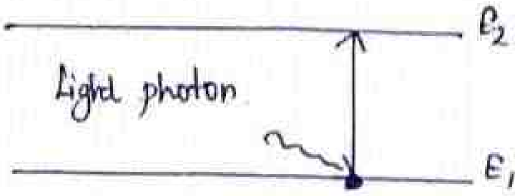
* This is known as optical pumping. (8)



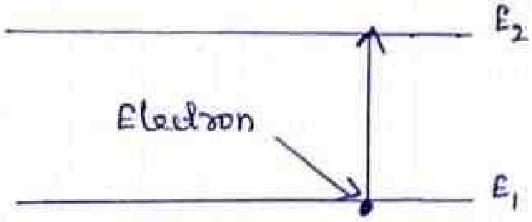
$E_2 > E_1$

This type of pumping is used in solid state lasers like ruby and Nd-YAG lasers.

ii) Electrical discharge :

The energy of the electrons is transferred to gas atoms. Thereby atoms again energy and go to excited state.

This results in population inversion. This is known as electrical discharge.



The energy transfer is represented by the eqn.

$$A + e^* \rightarrow A^* + e$$

Where, A - Gas atom in ground state.

A^* - Same gas atom in excited energy state.

e^* - Electron with more kinetic energy.

e - Same electron with less energy.

This method of pumping is used in gas lasers like argon and CO_2 lasers.

iii) Direct conversion

* In this method, the electrical energy is applied to a direct band gap semiconductor like Ga As.

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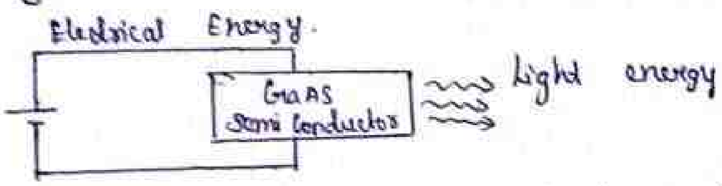
* The recombination of electrons and holes takes place. (9)

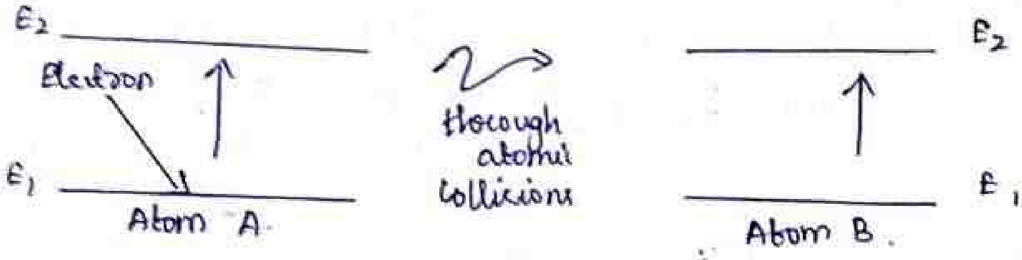
* During the recombination process, the electrical energy is directly converted into light energy.



This method of pumping is used in semiconductor diode laser.

iv) Inelastic collision between atoms:

* In this method, a combination of two gases is used.
* The excited energy levels of gases of A and B nearly coincides each other.



During the electrical discharge, atoms of gas A are excited to higher energy states A^* due to collision with the electrons.

$$A + e^* \rightarrow A^* + e$$

e^* — Electron with more kinetic energy.

e — same electron with less energy.

$$A^* + B \rightarrow A + B^*$$

Thus, population inversion in the energy states of B is achieved. This method is used in He-Ne laser.

15 Characteristics of Laser light:

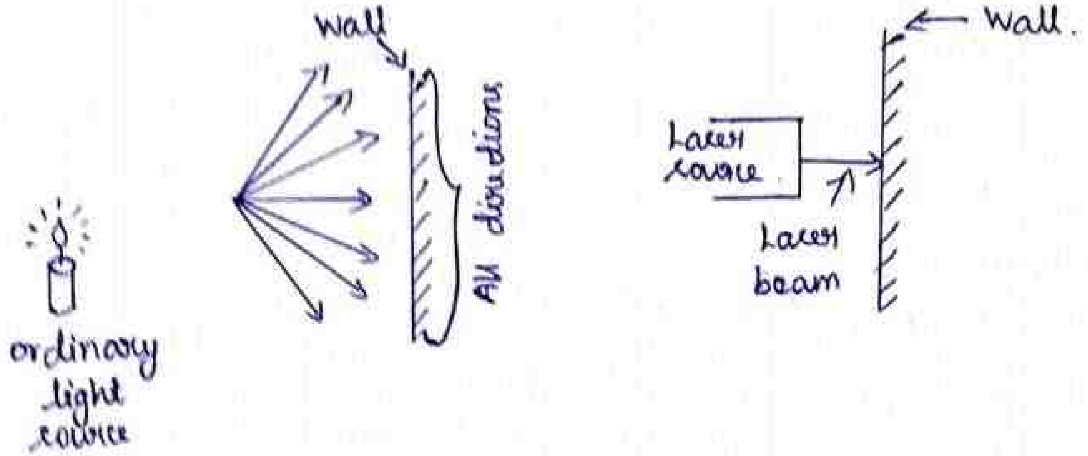
(u)

Laser is basically a light source. Laser light has the following important characteristics.

* High directionality
* High intensity.
* Highly monochromatic
* Highly coherent

(i) High directionality:

* An ordinary light source emits the light in all directions.
* But, a laser source emits light in only one direction.
* The divergence of laser beam is very small. So, laser light has high directionality.



ii) High Intensity:-

* Laser source emits light as a narrow beam and its energy is concentrated in a small region.
* This concentration of energy gives a high intensity to the laser light.

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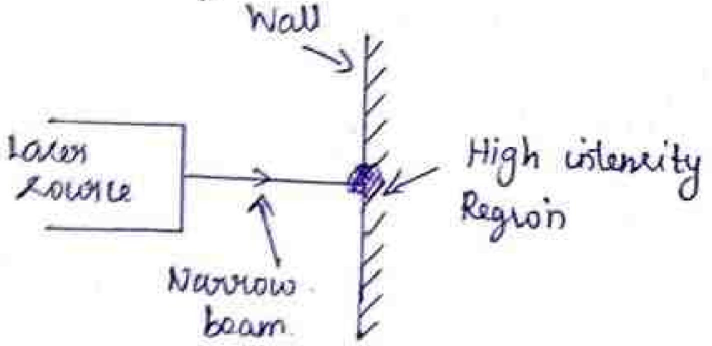
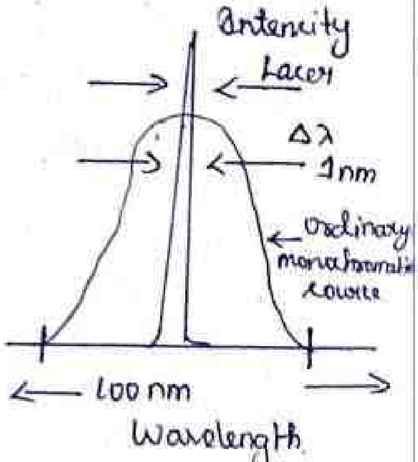
iii) Highly monochromatic:-

Ordinary light spreads over a wavelength range of the order of 100nm. ⑪

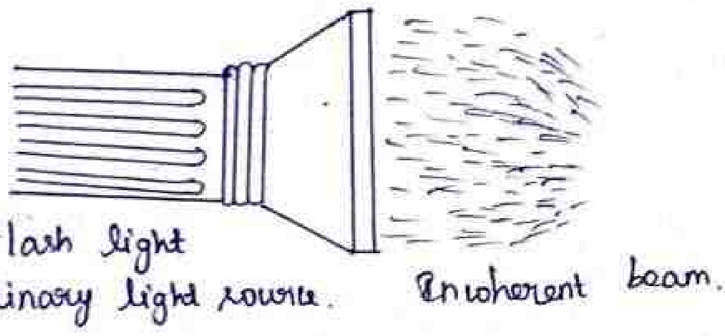


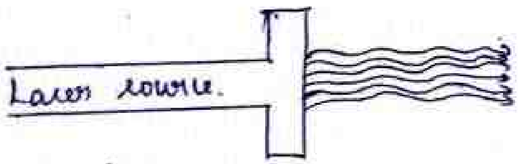


iv) Highly coherent:-

* The light emitted from a laser source consists of wave trains.

* Laser light has a high degree of coherence. The coherence of laser emission results in extremely high intensity and hence more power.





These important properties make the laser light superior to other conventional light sources such as flame, sunlight, ordinary electric bulbs, CFL, etc.

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Basic components of a laser system:-

A laser system consists of three important components.

* Active medium or material.
* pumping source.
* optical resonator.

a) Active medium or material:-

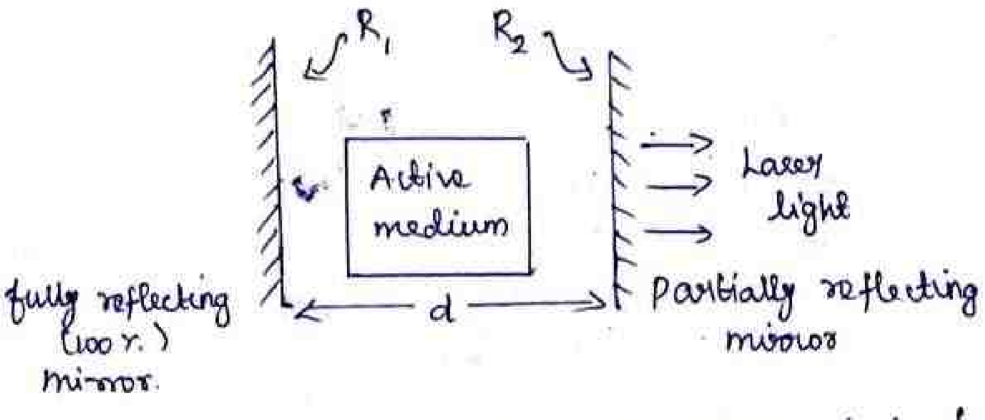
* It is a medium in which atomic transitions take place to produce laser action.
* The active medium may be a solid, liquid gas, dye or semiconductor.

b) pumping system:-

It is a system used to produce population inversion in the active medium.

c) optical resonator:-

The active medium is placed in b/n these two reflecting surfaces.



This induces more and more stimulated transition leading to laser action.

6. Nd. YAG LASER:-

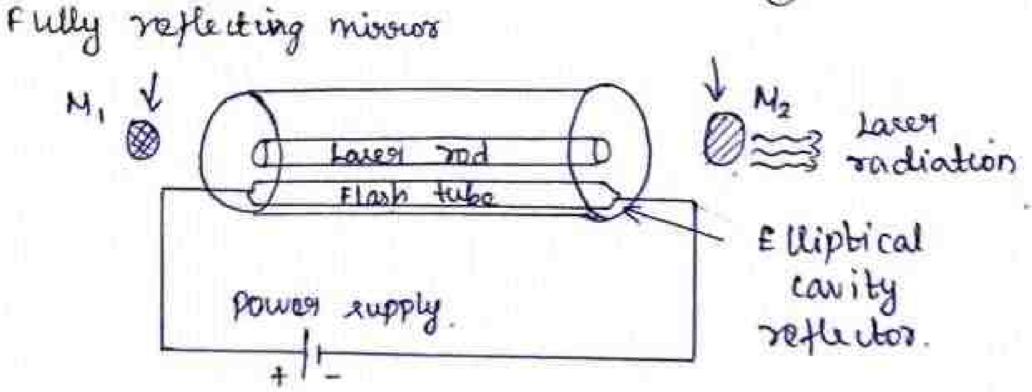
Nd-YAG Laser is Neodymium based laser. Nd stands for Neodymium and YAG for Yttrium Aluminium Garnet ($Y_3 Al_5 O_{12}$). It is a four level solid state laser.

Principle:-

* The active medium Nd-YAG rod is optically pumped by krypton flash tube.
* The neodymium ions (Nd^{3+}) are raised to excited energy levels.
* During transition from metastable state to ground state a laser beam of wavelength 1.064 μm is emitted.

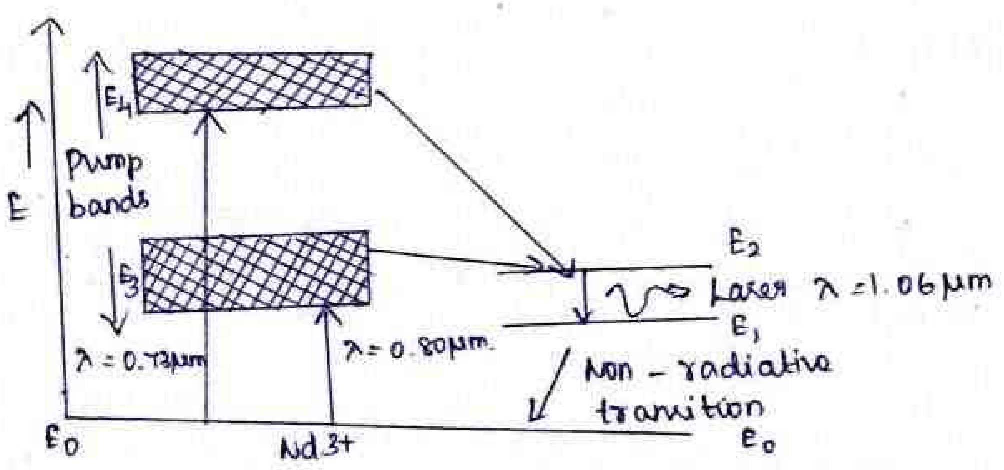
Construction:-

* A small amount of yttrium ions (Y^{3+}) is replaced with neodymium ions (Nd^{3+}) in the active medium of Nd-YAG rod.
* The active medium Nd-YAG crystal is cut into a cylindrical rod.
* The ends of this rod are highly polished and optically flat and parallel.
* The optical resonator is formed by using two external reflecting mirrors.
* One mirror M_1 is fully reflecting while the other mirror M_2 is partially reflecting.



Working:

* The energy level of Nd^{3+} ion in Nd-YAG laser.
* When krypton flash tube is switched on. the neodymium ions are excited from ground state E_0 to upper energy levels E_3 and E_4 due to absorption of light radiation of wavelengths 0.73μm and 0.80μm.
* The neodymium ions from there excited energy levels make a transition to energy level E_2 by non-radiative transition.
* E_2 is a metastable state.
* Now, the neodymium ions are collected in this energy level E_2.
* Thus. the population inversion is achieved b/n E_2 and E_1.



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Characteristics :-

Type :- It is a four - level solid, state laser.

Active medium : Nd - YAG rod.

pumping method : optical pumping.

pumping source : krypton flash tube

optical resonator : Two ends of Nd - YAG rod polished with silver.

power output : 20 KW.

Nature of output : pulsed or continuous beam of light.

wavelength of output : 1.06 µm.

Advantages :-

* This laser has high energy output.
* It is much easier to achieve population inversion.

Disadvantages :-

The electron energy level structure of Nd^{3+} in Nd-YAG is complicated.

Applications :-

* Nd - YAG laser is used in range finders and illuminators.
* It is widely used in resistor trimming, scribing micro - machining operations such as welding, drilling etc.
* It finds many medical applications such as endoscopy urology, neurosurgery.

Molecular Gas laser

In a molecular gas laser, laser action takes place by transitions b/n vibrational and rotational energy levels of gas molecules.

(16)

7. Carbon Dioxide (CO_2) Laser:-

* The first molecular CO_2 gas laser was developed by Indian born American scientist Prof. C.K.N. Patel.
* It is a four-level molecular gas laser.
* In this laser, transition takes place b/n vibrational energy states of carbon dioxide molecules.
* It is a very efficient laser.

Energy states of CO_2 molecules:-

* A carbon dioxide molecule has a carbon atom at the centre with two oxygen atoms. attached. one at each sides.
* such a molecule vibrates in three independent modes, They are.
    * symmetric stretching mode
    * Bending mode
    * Asymmetric stretching mode.

a) Symmetric stretching:

* In this mode of vibration, carbon atom is at rest.
* Both oxygen atoms vibrate such that they are moving away or approaching the fixed carbon simultaneously along the axis of the molecule.

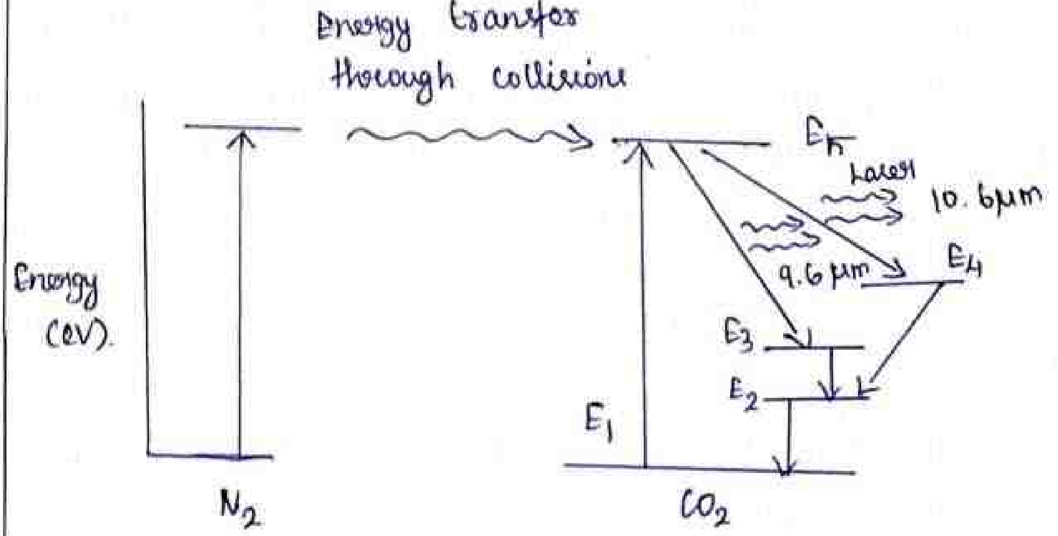
b) Bending:

In this mode of vibration. both oxygen atoms and carbon atom vibrate perpendicular to molecular axis.

Partially reflecting M_2. (19)

Working:

The energy level diagram of nitrogen and carbon dioxide molecules.



This process is represented by the eqn.

$$N_2 + e^* \rightarrow N_2^* + e$$

$N_2 \rightarrow$ Nitrogen molecule in ground state

$e^* \rightarrow$ Electron with high energy.

$N_2^* \rightarrow$ Nitrogen molecule in excited state

$e \rightarrow$ same electron with lesser energy.

This process is represented by the eqn.

$$N_2^* + CO_2 \rightarrow CO_2^* + N_2$$

$N_2^* \rightarrow$ Nitrogen molecule in excited state

$CO_2 \rightarrow$ carbon dioxide molecule in ground state.

$CO_2^* \rightarrow$ carbon dioxide molecule in excited state

$N_2 \rightarrow$ Nitrogen molecule in ground state.

There are two possible types of laser transition.

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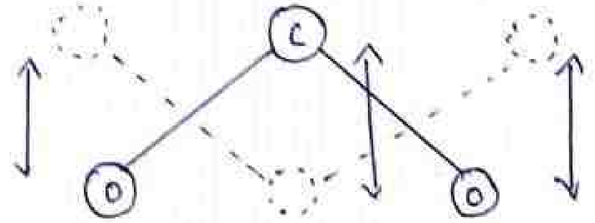
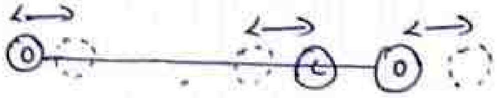
(18)



c) Asymmetric stretching :

In this mode of vibration, both oxygen atoms and carbon atom vibrate asymmetrically. oxygen atoms move in one direction while carbon atom moves in the opposite direction.



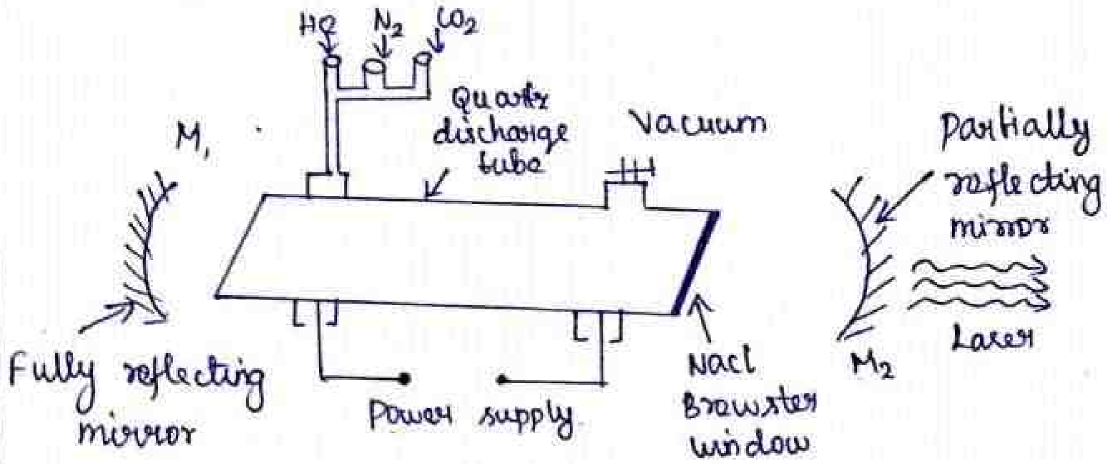
Principle

The laser transition takes place b/n the vibrational energy states of CO_2 molecules.

Construction:

* It consists of a quartz discharge tube 5m long and 2.5cm in diameter.

* This discharge tube is filled with the gas mixture of CO_2, nitrogen and helium with suitable partial pressures.



The optical resonator is formed with two concave mirrors one fully reflecting M_1 with the other

(19)

(i) Tranition $E_5 - E_4$

This tranition produces a laser beam of wavelength 10.6µm.

ii) Tranition $E_5 - E_3$

This tranition produces a laser beam of wavelength 9.6µm.

Normally 10.6µm tranition is more intense than 9.6 µm tranition. The power output from this laser is 10kW.

Characteristics:

Type : Molecular gas and four level laser

Active medium : A gas mixture of CO_2, N_2 and helium

pumping method: Electrical discharge method.

optical resonator: It is formed with two concave mirrors.

power output : 10 kW.

Nature of output: continuous wave or pulsed wave.

wavelength of output: 9.6µm & 10.6µm.

Advantages:

* The construction of CO_2 laser is simple.
* The output from this laser is continuous.
* It has high efficiency.
* It has very high output power.
* The output power can be increased by increasing the length of discharge tube.

Disadvantages:

* The contamination of oxygen by carbon monoxide has some effect on laser action.

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* corrosion may occur at the surfaces of the discharge tube. (20)

Applications:-

* It is used in remote sensing.
* It is used in the treatment of liver and lung diseases.
* It is mostly used in neurosurgery and general surgery.
* It is used to perform microsurgery and bloodless operations.

---

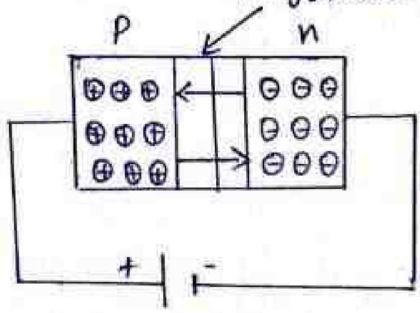
8. Semiconductor Laser:-

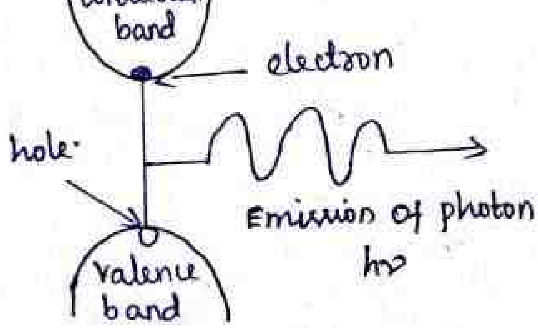
Definition:-

* It is a specially fabricated P-n junction diode.
* This diode emits laser light when it is forward-biased.

principle:-

* When the P-n junction diode is forward-biased
* The electrons from n-region and holes from p-region cross the junction and recombine with each other.
* This light radiation is known as recombination radiation.





Construction: (21)

* The construction of homo-junction semiconductor laser. is

* The active medium is a p-n junction diode made from a single crystal of gallium arsenide.

* This crystal is cut in the form of a platelet having a thickness of 0.5 mm.

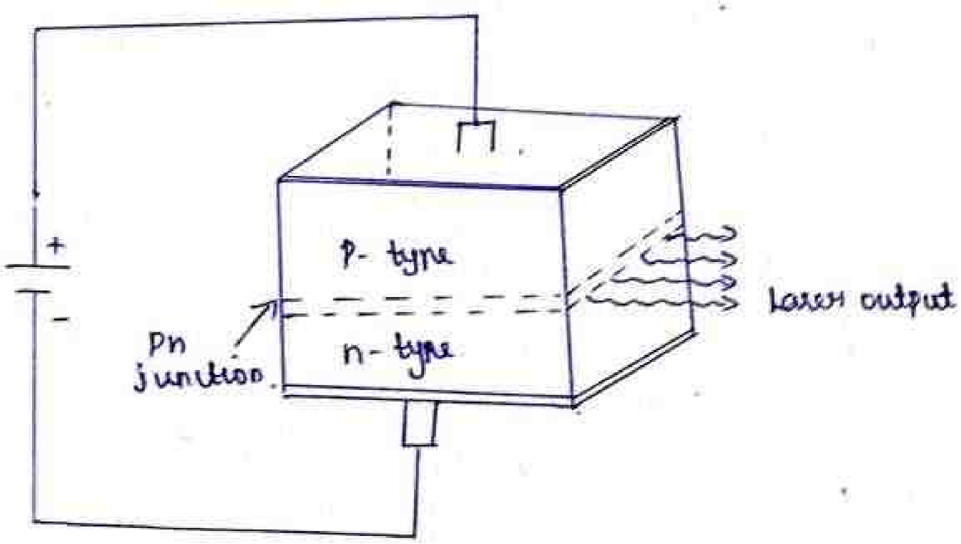
* This platelet consists of two regions n-type and p-type.

* The metal electrodes are connected to both upper and lower surfaces of the semiconductor diode.

* The forward bias voltage is applied through metal electrodes.

* Now the photon emission is stimulated in a very thin layer of Pn junction.

* They act as an optical resonator through which the emitted light comes out.



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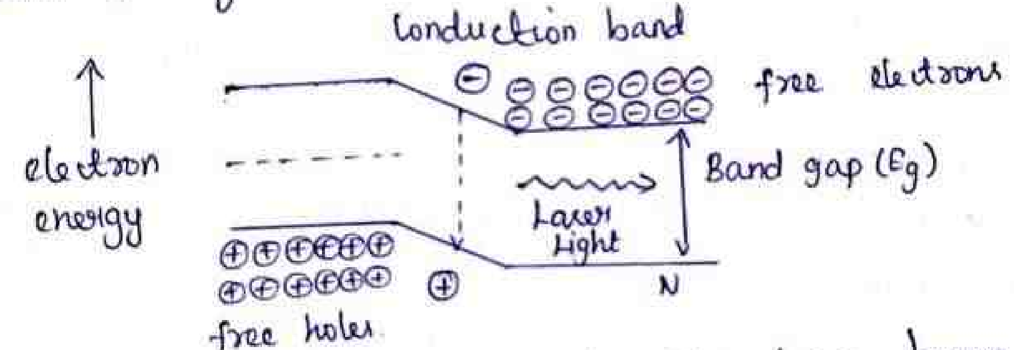
Working:-

* The energy level diagram of homojunction semiconductor laser.
* When the pn junction is forward-biased, the electrons and holes are injected into junction region.
* The region around junction contains a large number of electrons in the conduction band and holes in the valence band.
* Now the electrons and holes recombine with each other. During recombination, light photons are produced.



After gaining enough strength, laser beam of wavelength 8400Å is emitted from the junction.

The wavelength of laser light is given by.

$$E_g = h\nu = \frac{hc}{\lambda} \qquad \left(\because \nu = \frac{c}{\lambda}\right)$$

$$\lambda = \frac{hc}{E_g}$$

Where $E_g \rightarrow$ band gap energy in joule.

Characteristics:-

Type : Solid state semiconductor laser.

Active medium: A pn junction diode made from a single crystal of gallium arsenide.

Pumping method : Direct conversion method

Power output: a few mW.
Nature of output: continuous wave or pulsed output
wavelength of output: 8300 Å to 8500 Å.

Advantages:-

* This laser is very small in size and compact.
* It has high efficiency.
* It is operated with less power than ruby and CO_2 lasers.
* It requires very little additional equipment a
* It emits a continuous wave output or pulsed output.

Disadvantages:-

* Laser output beam has large divergence.
* The purity and monochromaticity are poor.
* It has poor coherence and stability.

Application:-

* This laser is widely used in fibre optic communication.
* It is used in laser printers and CD players.
* It is used to heal the wounds by infrared radiation.
* It is also used as a pain killer.

Semiconductor Laser (Hetero junction)

* A diode laser with a pn junction made up of different semiconductor materials in two regions.
* n-type and p-type is known as heterojunction semiconductor laser.

Principle:-

* when the pn jn diode is forward biased, the electrons from n-region and the holes from p-region

recombine with each other at the jn. (28)

* During recombination process, light photon is released.

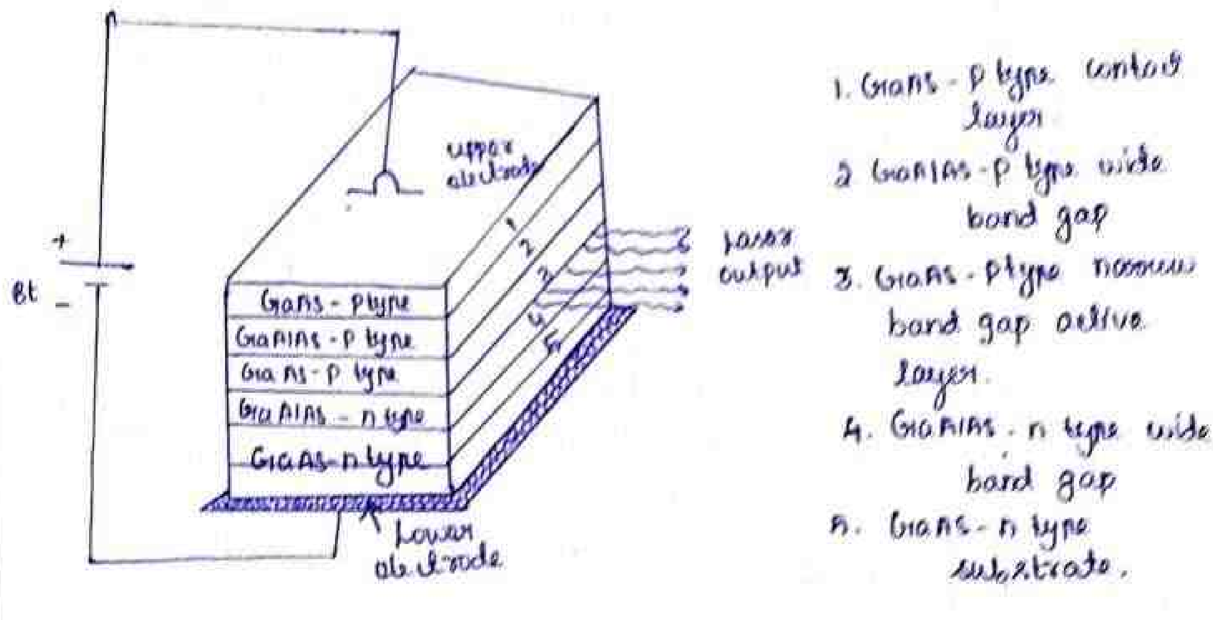
Example:-
Diode laser having a junction of GaAs and GaAlAs.

Construction:-

* A layer of GaAs p-type acts as active region. This layer is kept b/n two layers having wider band gap GaAlAs-p-type and GaAlAs n-type

* The end faces of the junctions of 3rd and 4th layers are well polished and parallel to each other.

* they act as an optical resonator.



9. Basic Applications of Laser in industry:-

Material processing

* Material processing involves cutting, welding, drilling and surface treatment.

* when the material is exposed to laser light, then light energy is converted into heat energy.

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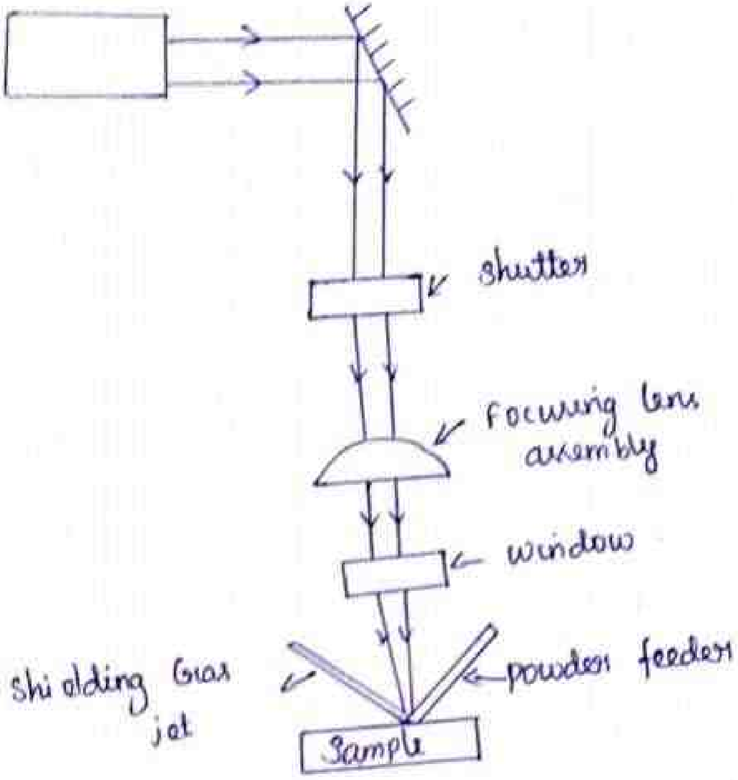
* Due to heating effect, the material is heated (5n) then melted and vapourised.

Laser instrumentation for material processing:

A Laser setup used for material process, such as Surface treatment, welding, cutting and drilling.



* The Shielding gas is used (i) to remove the molten material and help in vaporisation (ii) to provide cooling effect.

* For different materials, different gases are used.

Laser Annealing:

In annealing, there is no separate heat affected zone and melting takes place over few picometer thickness.

Laser hardening:

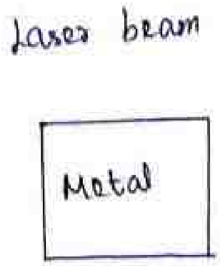
In hardening process, there is a heat affected zone in the form of hemisphere. 26

Laser surface alloying:

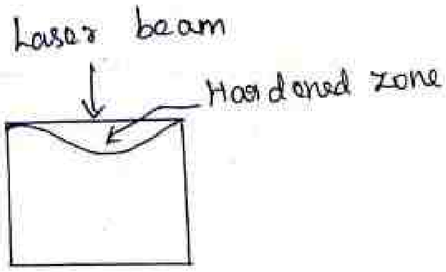
* Laser alloying involves the controlled melting of a work piece surface to a desired depth using laser.

Laser cladding:

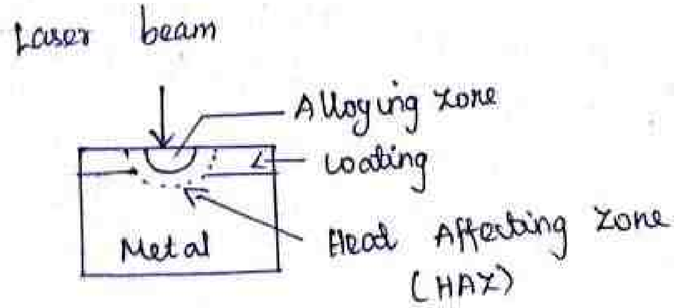
* In this process, a laser beam melts a very thin layer of work piece.
* This thin layer mixes with the liquid cladding alloy and form metallurgical bonding b/n the cladding and subtrate.



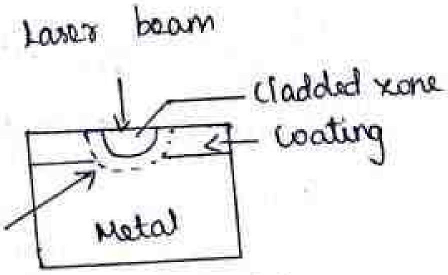
Annealing



Hardening



Surface alloying



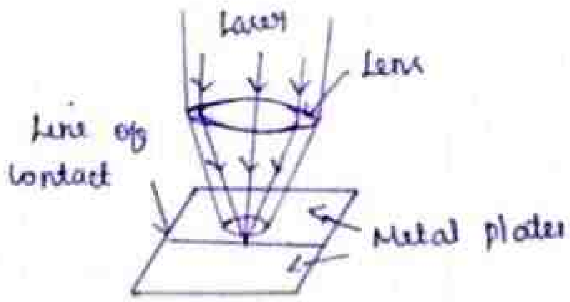
Surface cladding

Advantages:

* Heat treatment of metals using laser radiation is very fast.

10 ## Laser Welding:

* Welding is joining of two or more metal pieces into a single unit.
* The laser beam heats the edges of the two plates to their melting points.
* Metals fuse together where they are in contact.

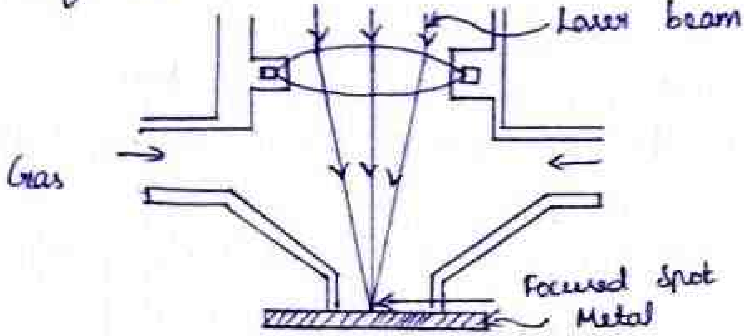


Advantages:

* It is a contact-less process and hence, there is no possibility of impurities into joint.
* Laser welding can be done even with very small pieces without any difficulty.
* The welding is done at very high rates.
* Any dissimilar metals can be welded.

Laser cutting (or) Drilling:

* The principle of laser cutting is the vaporization of the material at point of focus of the laser beam.
* The gas jet is also used to cool the adjacent edges of the cut metal.



Advantages:- (28)

* The microstructure of surrounding layers are not affected since heat affected zone is very narrow.

* Higher cutting speed can be achieved.

* Laser cutting can be done at room temperature and pressure without preheating and vacuum condition.

Soldering:-

* It is a process in which two or more metals are joined together by melting and putting a filler metal into the joint,

* The filler metal having a lower melting point than the adjoining metal.

* Laser soldering, the newest soldering method.

Laser soldering:-

* It is a process in which selectively heats solder by means of laser irradiation to form a bond b/n two parts.

Principle:-

* Laser soldering is a technique where a precisely focused laser beam provides controlled heating of the solder alloy leading to a fast and non-destructive of an electrical joint.

Working:-

* Laser soldering is a technique where a 30-50W laser is used to melt and solder an electrical connection joint.

* Both lead-tin and silver-tin material can be soldered.

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Laser soldering process:-

* The laser illuminate the soldering point.
* The illuminated area emits heat.
* solder is supplied.
* The heat transfers into the surrounding area and is raised to the melting temperature.

Types of lasers used in soldering:-

There are three main types of lasers are found suitable for soldering process. They are.

* carbon dioxide laser.
* Nd : YAG laser.
* semiconductor laser.

Advantages of laser soldering:-

In contrast to other conventional soldering techniques, laser soldering offers a lot of advantages. They include.

* High precision - spot sizes in the order of 100s of microns.
* Fast control of heat input
* It also has low maintenance.

Applications:-

The primary application for laser soldering is laser soldering of circuit boards in the electronics industry.

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Unit - IV ①

Basic Quantum Mechanics

1. Photons and Light waves - (Duality of Radiation and Matter)

The wave and particle duality of radiation is easily understood by knowing a difference between a wave and a particle.

wave:-

* A wave originates due to oscillations and it is spread out over a large region of space.
* A wave cannot be located at a particular place and mass cannot be carried by a wave.
* Actually, a wave is a spread out disturbance specified by its amplitude A, frequency ν, wavelength λ, phase δ and intensity I.
* The phenomena of interference, diffraction and polarisation require the presence of two or more waves at the same time and at the same position.

particle:-

* A particle is located at some definite point and it has mass. It can move from one place to another place.
* A particle is characterized by mass m, velocity ν, momentum p and energy E.
* Thus, radiation sometimes behaves as a wave and sometimes as a particle. Now, wave-particle duality of radiation is universally accepted.

(2)

2. Compton Effect :-

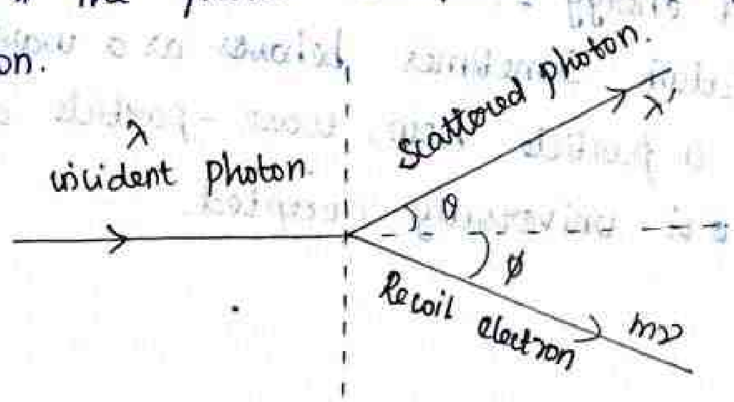
Compton effect reffers to the change in the wavelength of scattered x-rays by a material.

Statement :-

* when a beam of x-rays is scattered by a substance of low atomic number, the scattered x-ray radiation consists of two components.
* The change in the wavelength of scattered x-rays is known as compton shift. The phenomenon is called compton effect.
* the radiations of unchanged wavelength in the scattered radiations are called unmodified radiations.
* The radiations of longer wavelength are known as modified radiations.

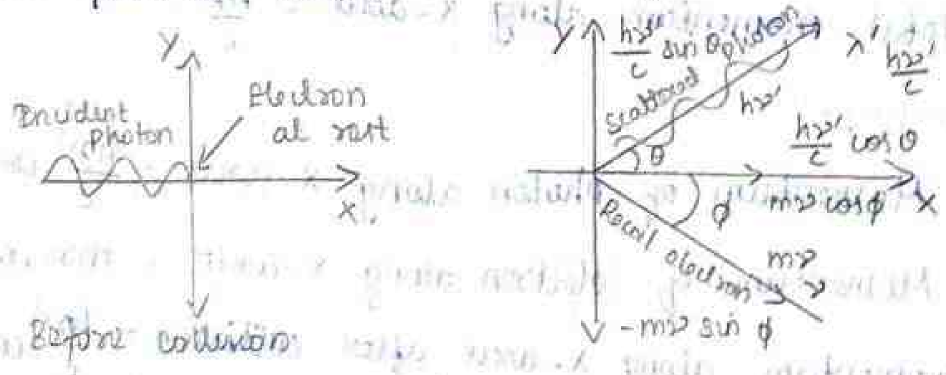
Explanation :-

* The compton effect was explained on the basis of quantum theory of radiation. The x-radiation consists of quanta or photons. each having an energy of $h\nu$.
* These photons move with velocity of light. (c).
* They obey the laws of conservations of energy and momentum when they undergo collision.
* the photon transfers some of its energy to the electron.



Theory of compton effect :- (Derivation)

Consider an x-ray photon striking an electron at rest. This x-ray photon is scattered through an angle θ to x-axis from its initial direction of motion.



Total energy before collision :-

Energy of incident photon $= h\nu$

Energy of electron at rest $= m_0 c^2$

where m_0 - rest mass of the electron,

c - velocity of light.

Total energy before collision $= h\nu + m_0c^2$

Total energy after collision :-

Energy of scattered photon $= h\nu'$

Energy of scattered electron $= mc^2$

where m is the mass of electron.

when it moves with velocity ν.

Total energy after collision $= h\nu' + mc^2$

Applying the law of conservation of energy.

Total energy before collision = Total energy after collision

$$h\nu + m_0c^2 = h\nu' + mc^2$$

$$mc^2 = h\nu - h\nu' + m_0c^2$$

$$\boxed{mc^2 = h(\nu - \nu') + m_0c^2} \quad \dots (1)$$

Total momentum along x-axis :- (Before collision)

Momentum of photon along x axis = $\frac{h\nu}{c}$

Momentum of electron along x-axis = 0

Total momentum along x-axis = $\frac{h\nu}{c}$

After collision :-

Momentum of photon along x-axis = $\frac{h\nu'}{c}\cos\theta$.

Momentum of electron along x-axis = $m\nu\cos\phi$.

Total momentum along x-axis after collision = $\frac{h\nu'}{c}\cos\theta + m\nu\cos\phi$.

Total momentum before collision = Total momentum after collision

$$\frac{h\nu}{c} = \frac{h\nu'}{c}\cos\theta + m\nu\cos\phi \quad \ldots (2)$$

$$\frac{h\nu}{c} - \frac{h\nu'}{c}\cos\theta = m\nu\cos\phi$$

$$\frac{h}{c}(\nu - \nu'\cos\theta) = m\nu\cos\phi$$

$$h(\nu - \nu'\cos\theta) = m\nu c\cos\phi.$$

$$m\nu c\cos\phi = h(\nu - \nu'\cos\theta) \quad \ldots (3)$$

Total momentum along y-axis.

Before collision :-

Momentum of photon along y-axis = 0.

Momentum of electron along y axis = 0.

Total momentum along y-axis = 0.

After collision :-

Momentum of photon along y-axis = $\frac{h\nu'}{c}\sin\theta$.

Momentum of electron along y-axis = $-m\nu\sin\phi$.

Total momentum along y-axis = $h\nu'/c\,\sin\theta - m\nu\sin\phi$.

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Applying the law of conservation of momentum. ⑤

Total momentum before collision = Total momentum after collision.

$$0 = \frac{h\nu'}{c}\sin\theta - m\nu\sin\phi$$

$$m\nu\sin\phi = \frac{h\nu'}{c}\sin\theta \quad \ldots (4)$$

$$m\nu c\sin\phi = h\nu'\sin\theta \quad \ldots (5)$$

Squaring eqn (3) and eqn (5) and then adding, we get.

$$(m\nu c\cos\phi)^2 + (m\nu c\sin\phi)^2 = h(\nu-\nu'\cos\theta)^2 + (h\nu'\sin\theta)^2 \quad \ldots (6)$$

L.H.S of eqn (6).

$$= m^2\nu^2c^2\cos^2\phi + m^2\nu^2c^2\sin^2\phi$$

$$= m^2\nu^2c^2(\sin^2\phi + \cos^2\phi) \qquad (\because \sin^2\phi + \cos^2\phi = 1)$$

$$= m^2\nu^2c^2$$

R.H.S of eqn (6).

$$= h^2(\nu^2 - 2\nu\nu'\cos\theta + \nu'^2\cos^2\theta) + h^2\nu'^2\sin^2\theta$$

$$= h^2[\nu^2 - 2\nu\nu'\cos\theta + \nu'^2\cos^2\theta + \nu'^2\sin^2\theta]$$

$$= h^2[\nu^2 - 2\nu\nu'\cos\theta + \nu'^2(\sin^2\theta + \cos^2\theta)]$$

$$= h^2(\nu^2 - 2\nu\nu'\cos\theta + \nu'^2) \qquad [\because \sin^2\theta + \cos^2\theta = 1]$$

L.H.S = R.H.S of eqn 6.

$$\boxed{m^2\nu^2c^2 = h^2(\nu^2 - 2\nu\nu'\cos\theta + \nu'^2)} \quad \ldots (7)$$

Squaring eqn (1), on both sides.

$$(mc^2)^2 = h(\nu-\nu') + m_0c^2)^2 \quad \ldots (8)$$

$$m^2c^4 = h^2(\nu-\nu')^2 + m_0^2c^4 + 2h(\nu-\nu')m_0c^2$$

$$\boxed{m^2c^4 = h^2(\nu^2 - 2\nu\nu' + \nu'^2) + 2h(\nu-\nu')m_0c^2 + m_0^2c^4} \quad \ldots (9)$$

Subtracting eqn (7) from eqn (9).

$$m^2c^4 - m^2\nu^2c^2 = h^2(\nu^2 - 2\nu\nu' + \nu'^2)^2 + 2h(\nu-\nu')m_0c^2 + m_0^2c^4 - h^2(\nu^2 - 2\nu\nu'\cos\theta + \nu'^2)$$

⑥

$$m^2c^2(c^2-\nu^2) = h^2\nu^2 - 2h^2\nu\nu' + h^2\nu'^2 + 2h(\nu-\nu')m_0c^2 + m_0^2c^4 - h^2\nu^2 + 2h^2\nu\nu'\cos\theta - h^2\nu'^2$$

$$m^2c^2(c^2-\nu^2) = -2h^2\nu\nu' + 2h(\nu-\nu')m_0c^2 + 2h^2\nu\nu'\cos\theta + m_0^2c^4$$

$$\boxed{m^2c^2(c^2-\nu^2) = -2h^2\nu\nu'(1-\cos\theta) + 2h(\nu-\nu')m_0c^2 + m_0c^4} \quad \ldots (10)$$

$$m = \frac{m_0}{\sqrt{\left(1-\frac{\nu^2}{c^2}\right)^2}} \quad \ldots (11)$$

squaring the eqn (11) on both sides, we have.

$$m^2 = \frac{m_0^2}{1-\frac{\nu^2}{c^2}} = \frac{m_0^2}{\frac{c^2-\nu^2}{c^2}} = \frac{m_0^2c^2}{c^2-\nu^2}$$

$$m^2(c^2-\nu^2) = m_0^2c^2$$

Multiplying c^2 on both sides, we have.

$$m^2c^2(c^2-\nu^2) = m_0^2c^2c^2$$

$$\boxed{m^2c^2(c^2-\nu^2) = m_0^2c^4} \quad \ldots (12)$$

Substituting eqn (12) in eqn (10), we get.

$$m_0^2c^4 = -2h^2\nu\nu'(1-\cos\theta) + 2h(\nu-\nu')m_0c^2 + m_0^2c^4$$

$$2h(\nu-\nu')m_0c^2 = 2h^2\nu\nu'(1-\cos\theta)$$

(or) $$\frac{\nu-\nu'}{\nu\nu'} = \frac{h}{m_0c^2}(1-\cos\theta)$$

$$\frac{\cancel{\nu}}{\cancel{\nu}\nu'} - \frac{\cancel{\nu'}}{\nu\cancel{\nu'}} = \frac{h}{m_0c^2}(1-\cos\theta)$$

$$\boxed{\frac{1}{\nu'} - \frac{1}{\nu} = \frac{h}{m_0c^2}(1-\cos\theta)} \quad \ldots (13)$$

Multiplying c on both sides of eqn (13).

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(7)

$$\frac{c}{\nu'} - \frac{c}{\nu} = \frac{hc}{m_0 c^2}(1-\cos\theta) \qquad \left[\because \frac{c}{\nu} = \lambda\right.$$

$$\frac{c}{\nu'} - \frac{c}{\nu} = \frac{h}{m_0 c}(1-\cos\theta) \qquad \left.\frac{c}{\nu'} = \lambda'\right]$$

$$\lambda' - \lambda = \frac{h}{m_0 c}(1-\cos\theta)$$

Therefore, the change in wavelength is given by,

$$\boxed{d\lambda = \frac{h}{m_0 c}(1-\cos\theta)} \quad \ldots (14)$$

It is found that the change in wavelength $(d\lambda)$ does not depend on the wavelength of the incident radiation.

Case (i). When $\theta = 0$ then,

$$d\lambda = \frac{h}{m_0 c}(1-\cos 0) \qquad (\because \cos 0 = 1)$$

$$d\lambda = \frac{h}{m_0 c}(1-1)$$

$$= \frac{h}{m_0 c} \times 0$$

$$\boxed{d\lambda = 0}$$

Along the incident direction, there is no change in wavelength.

Case (ii) When $\theta = 90°$, then

$$d\lambda = \frac{h}{m_0 c}(1-\cos 90°)$$

$$d\lambda = \frac{h}{m_0 c}(1-0) \qquad [\because \cos 90° = 0]$$

$$d\lambda = \frac{h}{m_0 c}$$

Substituting for h, m_0 and c, and have,

$$d\lambda = \frac{6.625 \times 10^{-34}}{9.11 \times 10^{-31} \times 3 \times 10^{8}}$$

$$\boxed{d\lambda = 0.0243 \text{ Å}}$$

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This difference in wavelength is known as compton (8)
wavelength of electron.

Case (iii) when $\theta = 180^\circ$, then

$$d\lambda = \frac{h}{m_0 c}(1-\cos 180^\circ) \qquad (\because \cos 180^\circ = -1)$$

$$d\lambda = \frac{h}{m_0 c}(1-(-1))$$

$$d\lambda = \frac{h}{m_0 c}(1+1) = \frac{2h}{m_0 c}$$

$$d\lambda = \frac{2h}{m_0 c}$$

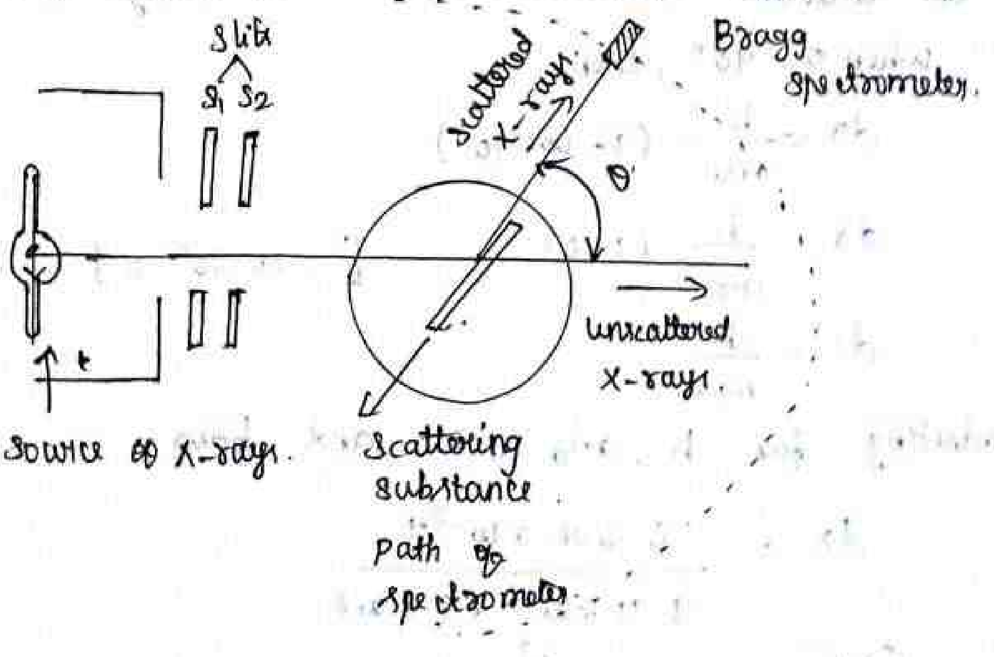
$$d\lambda = 2 \times 0.0243\,Å \qquad \left[\because \frac{h}{m_0 c} = 0.0243\,Å\right]$$

$$\boxed{d\lambda = 0.0486\,Å}$$

Thus, the change in wavelength is maximum at $\theta = 180^\circ$.

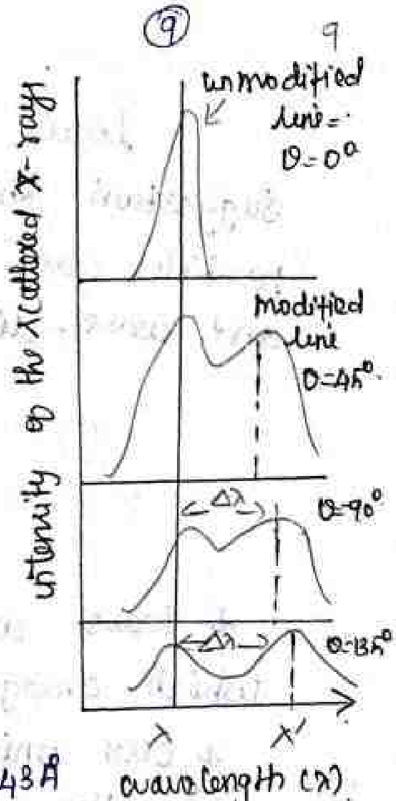
Experimental verification of compton effect:-

A beam of monochromatic x-rays of wavelength λ is made to incident on a scattering substance. The scattered x-rays are received by Bragg spectrometer.



It is found that the curves have two peaks, one corresponding to unmodified radiation and other corresponding to modified radiation.

The difference between two peaks on the wavelength axis gives compton shift.



$$\boxed{\lambda' - \lambda = d\lambda = \frac{h}{m_0 c}(1 - \cos\theta)}$$

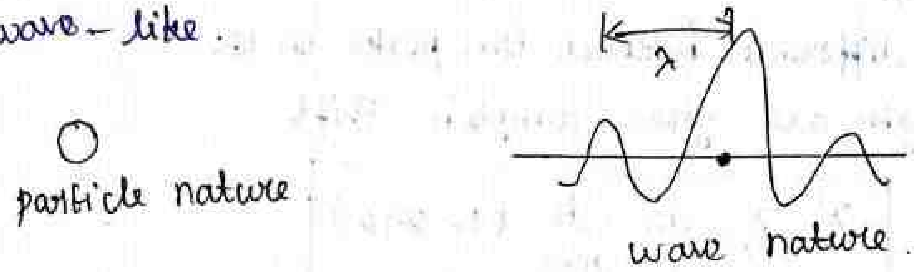
The change in wavelength $d\lambda = 0.0243$ Å at $\theta = 90^\circ$ is found to be in good agreement with the theoretical value 0.0243 Å. Thus, compton effect is experimentally verified.

3. Electrons (particles) And Matter waves - (concept of Matter waves).

* particle nature of matter is very well established. Now it is known that matter is composed of atoms, electrons, protons and neutrons.
* They are the building blocks of all types of atoms.
* on this background, in 1924 De Broglie extended the idea of dual nature of radiation to matter and proposed that matter possesses particle as well as wave characteristics.
* He believed that motion of electron within an atom is guided by a peculiar kind of waves called 'pilot waves'.
* while introducing the concept of matter waves. De-Broglie was guided by wave-particle duality of radiation and way in which nature manifests ~~herself~~ herself.

de-Broglie's Hypothesis:-

Louis de-Broglie proposed a very bold and novel suggestion that like light radiation; matter or material particle also possess dual characteristics (ie) particle-like and wave-like.



* waves and particles are the only two modes through which energy can propagate in nature.
* our universe is fully composed of light radiation and matter
* Since nature love symmetry, matter and waves must be symmetric
* Every moving particle is always associated with a wave.

de-Broglie waves and its wavelength:-

The waves associated with the matter particles are called matter waves or de-Broglie waves.

From the planck's theory, the energy of a photon of frequency ν is given by,

$$E = h\nu \quad \ldots (1)$$

According to Einstein's mass-energy relation

$$E = mc^2 \quad \ldots (2)$$

where m - mass of the photon.

c - velocity of the photon.

Equating (1) and (2), we get.

$$h\nu = mc^2 \quad \ldots (3)$$

$$\frac{hc}{\lambda} = mc^2 \qquad \left(\because \nu = \frac{c}{\lambda}\right)$$

$$\lambda = \frac{h\cancel{c}}{m c^{\cancel{2}}}$$

$$\lambda = \frac{h}{mc} \quad \text{(for electromagnetic radiation)}$$

Since $mc = p$ momentum of a photon.

Then $\lambda = \frac{h}{p}$ . . . (4).

Then momentum $p = m\nu$

$$\boxed{\lambda = \frac{h}{m\nu}} \quad \ldots (5)$$

This equation (5) is known as de-Broglie's wave equation.

<u>de-Broglie wavelength in terms of energy</u> :-

We know that the kinetic energy $E = \frac{1}{2}m\nu^2$

Multiplying by m on both sides we get,

$$mE = \frac{1}{2}m^2\nu^2 \quad \ldots (6).$$

$$2mE = m^2\nu^2$$

(or) $m^2\nu^2 = 2mE$.

Taking square root on both sides.

$$\sqrt{m^2\nu^2} = \sqrt{2mE}$$

$$m\nu = \sqrt{2mE}$$

Then $\lambda = \frac{h}{m\nu}$ . . . (7).

Substituting for $m\nu$ in eqn (7),

de-Broglie wavelength $\lambda = \frac{h}{\sqrt{2mE}}$

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de-Broglie's wavelength in terms of accelerating potential associated with electrons:-

When an electron of charge e is accelerated by a potential difference of V volts, then the electron gains a velocity v and hence,

workdone on the electron $= eV$ ... (1)

This workdone is converted into the kinetic energy of the electron as $\frac{1}{2}mv^2$

workdone = kinetic energy.

$$eV = \frac{1}{2}mv^2 \quad \ldots (2)$$

$$2eV = mv^2$$

$$mv^2 = 2eV$$

Multiply by m on both sides, we have,

$$m^2v^2 = 2meV$$

Taking square root on both sides, we get;

$$\sqrt{m^2v^2} = \sqrt{2meV}$$

$$mv = \sqrt{2meV} \quad \ldots (3)$$

From the de-Broglie concept, the wavelength associated with any moving particle is given by.

$$\lambda = \frac{h}{mv} \quad \ldots (4)$$

Substituting eqn (3) in eqn (4).

$$\boxed{\lambda = \frac{h}{\sqrt{2meV}}} \quad \ldots (5)$$

Substituting the given values, we have,

$h = 6.625 \times 10^{-34}$ JS, $e = 1.6 \times 10^{-19}$ C.

$m = 9.1 \times 10^{-31}$ kg.

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$$\lambda = \frac{6.625 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \times V}}$$

$$\lambda = \frac{12.25 \times 10^{-10}}{\sqrt{V}} \text{ metre} \quad \ldots (6)$$

$$\lambda = \frac{12.25}{\sqrt{V}} \text{ Å} \quad \ldots (7)$$

Properties of Matter waves :-

* If the mass of the particle is smaller, then the wavelength associated with that particle is longer.
* If the velocity of the particle is small, then the wavelength associated with that particle is longer.
* If $v = 0$, then $\lambda = \infty$, the wave becomes indeterminate and if $v = \infty$, then $\lambda = 0$.
* The velocity of de-Broglie waves is not constant since it depends on the velocity of the material particle.

Concept of wave function :-

In quantum mechanics it is postulated that there exists a function determined by the physical situation. The function is called wave function.

It is also postulated to contain all possible information about the system. Hence, it is also called as state function.

The total wave function can be represented by the equation.

$$\psi = Ae^{i(kx - \omega t)} \quad \ldots (1)$$

where A is a constant, ω is the angular frequency of the wave.

separating the space and time dependent parts, ψ and be expressed as,

$$\boxed{\psi = Ae^{ikx}\, e^{-i\omega t}} \quad \ldots (2)$$

In the above eqn, separating out the time dependent part,

$$\boxed{\psi = Ae^{ikx}} \quad \ldots (8)$$

ψ is time independent wave function. The total wave function is now written as,

$$\boxed{\Psi = \psi e^{-i\omega t}}$$

It is obtained by solving a fundamental equation called schrondinger equation.

* potential energy of the particle.
* Initial conditions, and,
* boundary conditions.

4. Schrodinger wave equation:-

* Schrodinger wave eqn describes the wave nature of particle in mathematical form. It is the basic equation of motion for matter waves.
* Schrodinger connected the expression of de-Broglie's wavelength with the classical wave equation for a moving particle.
* He obtained a new wave equation. This wave equation is known as schrodinger wave equation.

Forms of Schrodinger wave equations:-

There are two forms of schrodinger wave eqns. They are, (a) Time independent wave eqn.

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(b) Time dependent wave equation.

* one which is general and takes care of both the position and the time variations of the wave function, is called time - dependent Schrodinger equation.

* It involves the imaginary quantity 'i'

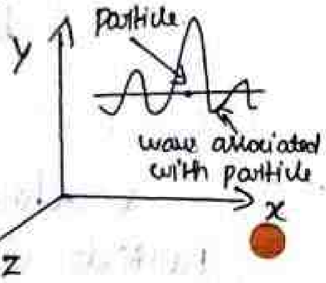
* The other one is applicable only to steady state conditions in which case, the wave function can have variation only with position but not with ~~not~~ time.

* It is called time - independent Schrodinger equation and is simpler than the other one. It doesn't involve it.

5. Schrodinger Time independent wave equation (Derivation)

* consider a wave associated with a moving particle.

* Let x, y, z be the coordinates of the particle and ψ wave function for de-Broglie's waves at any given instant of time t.



The classical differential eqn for wave motion is given by,

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} = \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2} \quad \text{---(1)}$$

here, v is wave velocity.

The eqn (1) is written as,

$$\nabla^2 \psi = \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2} \quad \text{--- (2)}$$

where, $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ is the laplacian's operator.

The solution of eqn (2) gives ψ as a periodic variation in terms of time t.

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(17)

$$\psi(x,y,z,t) = \psi_0(x,y,z)\, e^{-i\omega t}$$

$$\psi = \psi_0\, e^{-i\omega t} \quad \ldots (3)$$

Differentiating eqn (3) with respect to t, we get,

$$\frac{\partial \psi}{\partial t} = -i\omega\, \psi_0\, e^{-i\omega t}$$

Again differentiating with respect to t, we have,

$$\frac{\partial^2 \psi}{\partial t^2} = (-i\omega)(-i\omega)\, \psi_0\, e^{-i\omega t}$$

$$\frac{\partial^2 \psi}{\partial t^2} = i^2 \omega^2 \psi_0\, e^{-i\omega t}$$

$$\frac{\partial^2 \psi}{\partial t^2} = -\omega^2 \psi \quad \ldots (4) \qquad \left[\because i^2 = -1,\ \psi = \psi_0 e^{-i\omega t}\right]$$

Substituting eqn (4) in eqn (2)

$$\nabla^2 \psi = -\frac{\omega^2}{v^2}\psi$$

$$\nabla^2 \psi + \frac{\omega^2}{v^2}\psi = 0 \quad \ldots (5)$$

we know that angular frequency $\omega = 2\pi\nu = 2\pi\left(\frac{v}{\lambda}\right)$

here ν is the frequency

$$\frac{\omega}{v} = \frac{2\pi}{\lambda} \quad \ldots (6) \qquad \left(\because \nu = \frac{v}{\lambda}\right)$$

Squaring eqn (6) on both sides, we get,

$$\frac{\omega^2}{v^2} = \frac{2^2\pi^2}{\lambda^2} = \frac{4\pi^2}{\lambda^2} \quad \ldots (7)$$

substituting eqn (7) in eqn (5), we have,

$$\nabla^2 \psi + \frac{4\pi^2}{\lambda^2}\psi = 0 \quad \ldots (8)$$

on substituting $\lambda = \frac{h}{mv}$ in eqn (8).

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(18)

$$\nabla^2\psi + \frac{4\pi^2}{\frac{h^2}{m^2 v^2}}\psi = 0$$

$$\nabla^2\psi + \frac{4\pi^2 m^2 v^2}{h^2}\psi = 0 \quad \ldots (9)$$

If E is total energy of the particle, V is potential energy and $\frac{1}{2}mv^2$ is kinetic energy, then,

Total energy = potential energy + kinetic energy

$$E = V + \frac{1}{2}mv^2$$

or $$E - V = \frac{1}{2}mv^2$$

$$2(E-V) = mv^2$$

$$mv^2 = 2(E-V)$$

Multiplying by m on both sides,

$$m^2v^2 = 2m(E-V) \quad \ldots (10).$$

Substituting eqn (10) in eqn (9).

$$\nabla^2\psi + \frac{4\pi^2}{h^2} \times 2m(E-V)\psi = 0.$$

$$\boxed{\nabla^2\psi + \frac{8\pi^2}{h^2}(E-V)\psi = 0} \quad \ldots (11).$$

The eqn (11) is known as Schrodinger time independent wave equation for ~~f~~ three dimensions.

$$\hbar = \frac{h}{2\pi} \text{ in eqn (11).}$$

$$\hbar^2 = \frac{h^2}{2^2\pi^2} = \frac{h^2}{4\pi^2} \quad \ldots (12).$$

where $\hbar$ is a reduced planck's constant.

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The eqn (11) is modified by substituting $\hbar$.

$$\nabla^2\psi + \frac{m}{\frac{h^2}{8\pi^2}}(E-V)\psi = 0$$

$$\nabla^2\psi + \frac{m}{\frac{\hbar^2}{2\times 2^2\pi^2}}(E-V)\psi = 0$$

$$\nabla^2\psi + \frac{2m}{\frac{\hbar^2}{4\pi^2}}(E-V)\psi = 0 \quad \ldots (13)$$

On substituting eqn (12) in eqn (13), Schrodinger's time independent wave eqn is written as,

$$\boxed{\nabla^2\psi + \frac{2m}{\hbar^2}(E-V)\psi = 0} \quad \ldots (14)$$

or

$$\boxed{-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = E\psi} \quad \ldots (15)$$

Special case :-

If we consider one-dimensional motion i.e, particle moving along x-direction, then schrodinger time independent equation (14) reduces to,

$$\boxed{\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E-V)\psi = 0} \quad \ldots (16)$$

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6. Schrodinger Time dependent wave equation :- (20)

Schrodinger time dependent wave equation is derived from schrodinger time independent wave equation.

The solution of classical differential equation of wave motion is given by.

$$\psi(x, y, z, t) = \psi_0 (x, y, z) e^{-i\omega t} \quad \ldots (1)$$

Differentiating eqn (1) with respect to time t,

$$\frac{\partial \psi}{\partial t} = -i\omega \psi_0 e^{-i\omega t} \quad \ldots (2)$$

$$\frac{\partial \psi}{\partial t} = -i(2\pi\nu)\psi_0 e^{-i\omega t} \qquad (\because \omega = 2\pi\nu)$$

$$\frac{\partial \psi}{\partial t} = -2\pi i \nu \psi \quad \ldots (3) \qquad (\because \psi = \psi_0 e^{-i\omega t})$$

$$\frac{\partial \psi}{\partial t} = -2\pi i \frac{E}{h} \psi \qquad \left(\because E = h\nu \text{ or } \nu = \frac{E}{h}\right)$$

$$\frac{\partial \psi}{\partial t} = -i \frac{E}{\frac{h}{2\pi}} \psi = -i \frac{E}{\hbar} \psi \qquad \left[\because \hbar = \frac{h}{2\pi}\right]$$

$$\frac{\partial \psi}{\partial t} = -i \frac{E}{\hbar} \psi \quad \ldots (4)$$

Multiplying i on the both sides in eqn (4)

$$i \frac{\partial \psi}{\partial t} = -i \times i \left(\frac{E}{\hbar}\right) \psi = -i^2 \left(\frac{E}{\hbar}\right) \psi$$

$$i \frac{\partial \psi}{\partial t} = \frac{E}{\hbar} \psi \qquad [\because i \times i = i^2 = -1]$$

$$\hbar i \frac{\partial \psi}{\partial t} = E\psi \quad \ldots (5)$$

schrodinger time independent wave eqn is,

$$\boxed{-\frac{\hbar^2}{2m} \nabla^2 \psi + V\psi = E\psi}$$

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Substituting for $E\psi$ from eqn (π).

$$-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = i\hbar\frac{\partial\psi}{\partial t}$$

$$\left(-\frac{\hbar^2}{2m}\nabla^2 + V\right)\psi = i\hbar\frac{\partial\psi}{\partial t} \quad \ldots (4)$$

or) $H\psi = E\psi$

where $H = \left(-\frac{\hbar^2}{2m}\nabla^2 + V\right)$ is Hamiltonian operator,

$E = i\hbar\frac{\partial}{\partial t}$ is energy operator.

This eqn (4) is known as Schrodinger time dependent wave equation.

---

7. Meaning or physical significance of wave function ψ.

* The variable quantity which describes de-Broglie wave is called wave function ψ.

* It connects the particle nature and its associated wave nature statistically.

* The probability 0 corresponds to the certainty of not finding the particle and probability 1 corresponds to certainty of finding the particle.

i.e. $\iiint \psi^* \psi \, d\tau = 1$, if particle is present.

$= 0$ if particle is not present.

where ψ^* = complex conjucate of ψ.

* The probability of finding a particle at a particular region must be real and positive, but the wave function ψ is in general a complex quantity.

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8. Motion of A free particle:-

(22)

Let us consider electrons propagating freely in space in the positive x-direction and not acted upon by any force.

As the electrons are not acted upon by any force, their potential energy V is zero. Schrodinger equation,

$$\nabla^2 \psi + \frac{8\pi^2 m}{h^2}(E - V)\psi = 0$$

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2} E\psi = 0 \quad \ldots (1)$$

Taking $\frac{8\pi^2 mE}{h^2} = k^2$ in the above equation, we get,

$$\frac{d^2\psi}{dx^2} + k^2\psi = 0.$$

The general solution of the above equation is,

$$\psi(x) = Ae^{ikx} + Be^{-ikx}$$

$$\psi(x,t) = Ae^{ikx} e^{-i\omega t}$$

$$E = \frac{h^2 k^2}{8\pi^2 m} \quad \ldots (2)$$

A freely moving electron therefore possess a continuous energy spectrum as,

It is noted from equation (2) that,

$$k = \sqrt{\frac{2mE}{\hbar^2}} \qquad (\because P = \sqrt{2mE})$$

$$k = \sqrt{\frac{2mE}{\hbar^2}} = \frac{P}{\hbar}$$

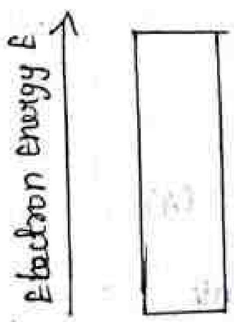
(23)

$$k = \frac{p}{\frac{h}{2\pi}} = \frac{2\pi p}{h} = \frac{2\pi}{\lambda} \qquad \left(\frac{p}{h} = \frac{1}{\lambda}\right)$$

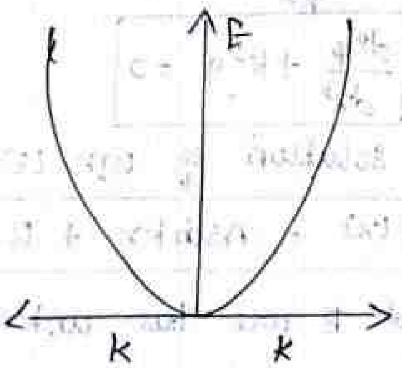
The 'k' known as wave vector describes the wave properties of the electrons. further. it is seen from the relation (2) that

$$E \propto k^2$$

The plot of E as a function of k gives a parabola as explained, in fig.



The energy continuum of free electron.



The parabolic relationship between energy and E and wave vector k in case of a free electron.

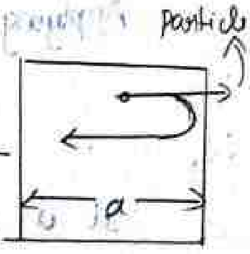
9. Particle in a infinite potential :- (one-Dimensional Box).

The walls are infinitely high. The potential energy V of the particle is infinite outside the walls

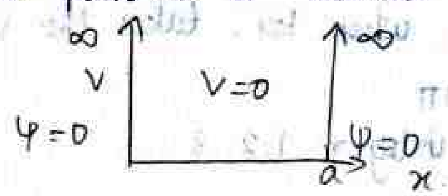
Thus the potential function is given by,

$$\boxed{\begin{array}{l} V(x) = 0 \text{ for } 0 < x < a. \\ V(x) = \infty \text{ for } 0 \geq x \geq a \end{array}}$$

Box. ←



This potential function is known as square well potential.



Now, task is to find the value ψ within the box i.e. between $x=0$ and $x=a$.

Schroedinger's wave equation in one-dimension is given by.

$$\boxed{\frac{\partial^2\psi}{\partial x^2}+\frac{2m}{\hbar^2}(E-V)\psi=0} \quad \ldots (1)$$

Since $V=0$, eqn (1) reduces to

$$\frac{d^2\psi}{dx^2}+\frac{2mE}{\hbar^2}\psi=0 \quad \ldots (2)$$

Substituting $\frac{2mE}{\hbar^2}=k^2$ in eqn (2).

$$\boxed{\frac{d^2\psi}{dx^2}+k^2\psi=0} \quad \ldots (3)$$

The general solution of eqn (3) is give by.

$$\boxed{\psi(x) = A\sin kx + B\cos kx} \quad \ldots (4)$$

here A and B are two unknown constants.

<u>Boundary condition (i)</u>

$\psi=0$ at $x=0$.

Applying this condition to eqn (4).

$$0 = A\sin 0 + B\cos 0 \qquad \left[\because \sin 0 = 0,\ \cos 0 = 1\right]$$

$$= 0 + B\times 1 \Rightarrow B=0$$

<u>Boundary condition (ii)</u>.

$\psi=0$ at $x=a$.

Applying this condition to eqn (4), we have.

$$0 = A\sin ka + 0 \qquad [\because B=0]$$

$$A\sin ka = 0.$$

It is found that either $A=0$ or $\sin ka=0$.

$$\therefore \sin ka = 0$$

$\sin ka$ is '0' only when ka takes the value of $n\pi$

$$ka = n\pi$$

where n is positive integer 1, 2, 3 ....

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$$k = \frac{n\pi}{a} \quad \dots (5).$$

on squaring eqn (5) we have.

$$k^2 = \frac{n^2\pi^2}{a^2} \quad \dots (6).$$

we know that $k^2 = \frac{2mE}{\hbar^2} = \frac{2mE}{\frac{h^2}{4\pi^2}}$ $\left[\because \hbar = \frac{h}{2\pi}\right]$

$$k^2 = \frac{(2m \times 4\pi^2)E}{h^2}$$

$$k^2 = \frac{8\pi^2 mE}{h^2} \quad \dots (7).$$

Equating eqn (6) and eqn (7).

$$\frac{n^2\pi^2}{a^2} = \frac{8\pi^2 mE}{h^2}$$

Energy of the particle.

$$E_n = \frac{n^2 h^2}{8ma^2} \quad \dots (8)$$

Substituting eqn (5) in eqn (4).

$$\psi_n(x) = A\sin \frac{n\pi x}{a} \quad \dots (9).$$

$n = 1, 2, 3 \dots$

for each value of n. there is an energy level.

10. Normalisation of wave function:

The constant A is determined by normalisation of wave function as follows.

Probability density is given by $\psi^*\psi$.

we know that $\psi_n(x) = A\sin \frac{n\pi x}{a}$.

$$\psi^*\psi = A\sin \frac{n\pi x}{a} \times A\sin \frac{n\pi x}{a}.$$

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$$\psi^* \psi = A^2 \sin^2 \left[\frac{n\pi x}{a}\right] \quad \ldots (10)$$

$$\int_0^a \psi^* \psi \, dx = 1 \quad \ldots (11)$$

Substituting $\psi^* \psi$ from eqn (10) in eqn (11).

$$\int_0^a A^2 \sin^2 \left(\frac{n\pi x}{a}\right) dx = 1$$

$$A^2 \int_0^a \left(\frac{1 - \cos\left(\frac{2n\pi x}{a}\right)}{2}\right) dx = 1 \quad \left(\because \sin^2\theta = \frac{1-\cos 2\theta}{2}\right)$$

$$\frac{A^2}{2}\left[\int_0^a dx - \int_0^a \cos\left(\frac{2n\pi x}{a}\right) dx\right] = 1.$$

The second term of the integral becomes zero at both time,

$$\frac{A^2}{2}[x]_0^a = 1$$

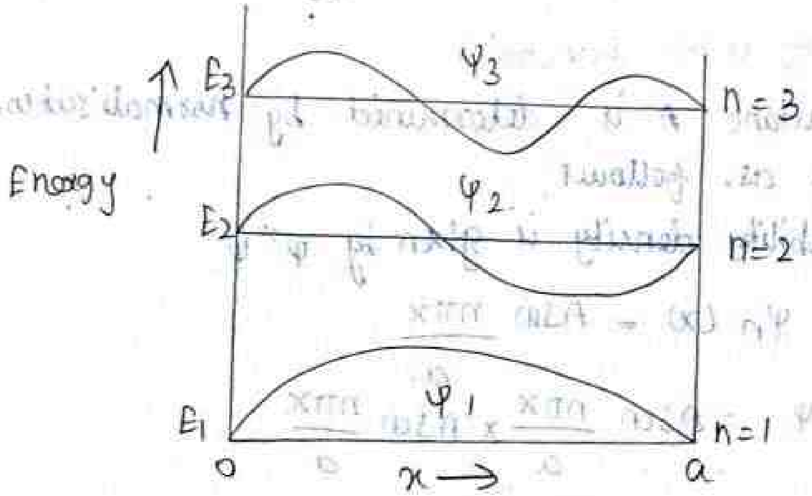
$$\frac{A^2 a}{2} = 1 \text{ or } A^2 = \frac{2}{a}.$$

$$A = \sqrt{\frac{2}{a}} \quad \ldots (12)$$

on substituting eqn (12) in eqn (9), we have,

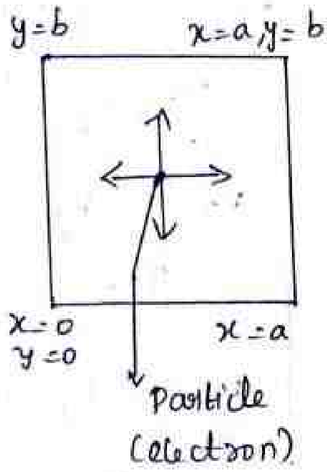
$$\psi_n = \sqrt{\frac{2}{a}} \sin \frac{n\pi x}{a} \quad \ldots (13).$$

This expression (13) is known as normalised eigen function.



11. ## Extension To Two Dimensions (2D Boxes)

* The solution of one-Dimensional potential well is extended for a two-dimensional potential well.
* In a two-dimensional potential well, the particle can freely move in two directions (x and y).
* we have to use two quantum numbers, n_x and n_y corresponding to the two coordinate axes namely x and y respectively.



Energy of the particle $E = E_{n_x} + E_{n_y}$

$$E_{n_x n_y} = \frac{n_x^2 h^2}{8ma^2} + \frac{n_y^2 h^2}{8mb^2}$$

$$a = b$$

$$E_{n_x n_y} = \frac{h^2}{8m}\left[\frac{n_x^2}{a^2} + \frac{n_y^2}{a^2}\right]$$

$$E_{n_x n_y} = \frac{h^2}{8ma^2}\left[n_x^2 + n_y^2\right] \quad \dots (1)$$

The corresponding normalised wave function of the particle in the two dimensional well is written as,

$$\psi_{n_x n_y} = \sqrt{\frac{2}{a}} \sin\left(\frac{n_x \pi x}{a}\right) \times \sqrt{\frac{2}{b}} \sin\left(\frac{n_y \pi y}{b}\right)$$

$$\psi_{n_x n_y} = \sqrt{\frac{4}{ab}} \sin\left(\frac{n_x \pi x}{a}\right) \sin\left(\frac{n_y \pi y}{b}\right) \quad \dots (2)$$

From equations (1) and (2),
we understand that several combinations of the two quantum numbers lead to different energy eigen values and eigen functions.

Example

Suppose a state has quantum numbers,

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$n_x = 1, n_y = 2,$

$n_x^2 + n_y^2 = 1^2 + 2^2 = 1 + 4 = 5,$

$n_x = 2, n_y = 1$

$n_x^2 + n_y^2 = 2^2 + 1^2 = 4 + 1 = 5;$

$$E_{12} = E_{21} = \frac{5h^2}{8ma^2} \quad \cdots (3)$$

The corresponding wave functions is written as,

$$\psi_{n_x n_y} = \sqrt{\frac{4}{ab}} \left\{ \sin\left(\frac{n_x \pi x}{a}\right) \sin\left(\frac{n_y \pi y}{b}\right) \right.$$

$n_x = 1, n_y = 2.$

$$\psi_{12} = \sqrt{\frac{4}{ab}} \left\{ \sin\left(\frac{\pi x}{a}\right) \sin\left(\frac{2\pi y}{b}\right) \right.$$

$n_x = 2, n_y = 1$

$$\psi_{21} = \sqrt{\frac{4}{ab}} \left\{ \sin\left(\frac{2\pi x}{a}\right) \sin\left(\frac{\pi y}{b}\right) \right. \cdots (4)$$

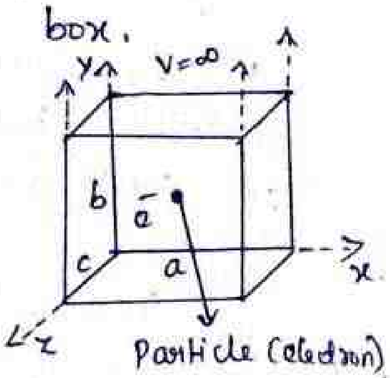
12. Extension To infinite well Three Dimensions (3D Box).

* The solution of one-dimensional potential well is extended for a three-dimensional (3D) potential box.
* we have to use three quantum numbers, n_x, n_y and n_z, corresponding to the three coordinate axes namely x, y and z respectively.
* If a, b, c are the lengths of the box.

Energy of the particle $= E_x + E_y + E_z$

$$E_{n_x n_y n_z} = \frac{n_x^2 h}{8ma^2} + \frac{n_y^2 h}{8mb^2} + \frac{n_z^2 h}{8mc^2}$$

If $a = b = c$ as for a cubical box,



$$E_{n_x n_y n_z} = \frac{h^2}{8m}\left[\frac{n_x^2}{a^2} + \frac{n_y^2}{a^2} + \frac{n_z^2}{a^2}\right]$$

$$E_{n_x n_y n_z} = \frac{h^2}{8ma^2}\left[n_x^2 + n_y^2 + n_z^2\right] \quad \ldots (1)$$

The corresponding normalised wave function of the particle in the three dimension well is written as,

$$\psi_{n_x n_y n_z} = \sqrt{\frac{2}{a}} \sin\left(\frac{n_x \pi x}{a}\right) \sqrt{\frac{2}{b}} \sin\left(\frac{n_y \pi y}{b}\right) \sqrt{\frac{2}{b}} \sin\left(\frac{n_z \pi z}{c}\right)$$

$$\psi_{n_x n_y n_z} = \sqrt{\frac{8}{abc}} \sin\left(\frac{n_x \pi x}{a}\right) \sin\left(\frac{n_y \pi y}{b}\right) \sin\left(\frac{n_z \pi z}{c}\right) \ldots (2)$$

From equations (1) and (2), we understand that several combinations of the three quantum numbers $(n_x\, n_y\, n_z)$ lead to different energy eigen values and eigen functions.

Example

$$n_x = 1, \; n_y = 1, \; n_z = 2.$$

$$n_x^2 + n_y^2 + n_z^2 = 1^2 + 1^2 + 2^2 = 1+1+4 = 6$$

$$n_x = 1, \; n_y = 2, \; n_z = 1$$

$$n_x^2 + n_y^2 + n_z^2 = 1^2 + 2^2 + 1^2 = 1+4+1 = 6$$

$$n_x = 2, \; n_y = 1, \; n_z = 1$$

$$n_x^2 + n_y^2 + n_z^2 = 2^2 + 1^2 + 1^2 = 4+1+1 = 6$$

$$\therefore E_{112} = E_{121} = E_{211} = \frac{6h^2}{8ma^2} \quad \ldots (3)$$

The corresponding wave functions is written as,

$$\psi_{112} = \sqrt{\frac{8}{a^3}} \sin\frac{\pi x}{a} \sin\frac{\pi y}{b} \sin\frac{2\pi z}{c}$$

(30)

$$\left.\begin{aligned} \psi_{121} &= \sqrt{\frac{8}{a^3}} \sin \frac{\pi x}{a} \sin \frac{2\pi y}{b} \sin \frac{\pi z}{c} \\ \psi_{211} &= \sqrt{\frac{8}{a^3}} \sin \frac{2\pi x}{a} \sin \frac{\pi y}{b} \sin \frac{\pi z}{c}. \end{aligned}\right\} \quad \cdots (4)$$

Degeneracy.

* It is noted from eqns (3) and (4) that, for several combinations of quantum numbers, we
* we have the same energy eigen value but different eigen functions,
* such a state of energy levels is called degenerate state.
* The three combinations of quantum numbers, (112), (121), (211)

Non-degenerate state:

when only one wave function corresponds to the energy eigen value, such a state is called non-degenerate state.

$n_x = 2, \; n_y = 2, \; n_z = 2.$

$$E_{222} = \frac{12h^2}{8ma^2}$$

$$\psi_{222} = \sqrt{\frac{8}{a^3}} \sin \frac{2\pi x}{a} \sin \frac{2\pi y}{b} \sin \frac{2\pi z}{a}$$

13. **Probability Density.**

probability of finding the particle between positions x and $x + dx$.

$$p(x) = |\psi_n|^2 \, dx$$

$$= \frac{2}{a} \sin \left(\frac{n\pi x}{a}\right) dx.$$

∴ probability density, $p(x) = \frac{2}{a}\sin^2\left(\frac{n\pi x}{a}\right)$ (31)

probability density is maximum,

$$\frac{n\pi x}{a} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$$

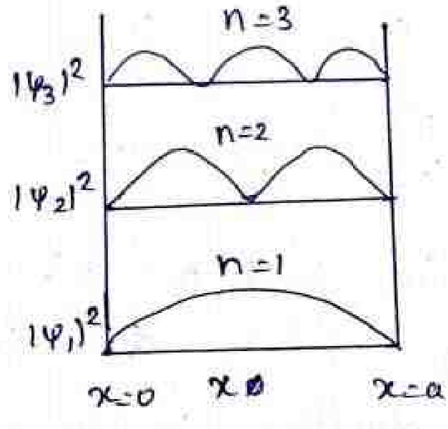
$$x = \frac{a}{2n}, \frac{3a}{2n}, \frac{5a}{2n}, \ldots$$

* For $n=1$, $x = \frac{a}{2}$, the particle is most likely to be in the middle of the box.

* For $n=2$, $x = \frac{a}{4}$ and $\frac{3a}{4}$, the particle is most likely to be at $\frac{a}{4}$ and $\frac{3a}{4}$ and never found in the middle because $|\psi_2|^2$ is zero.

* For $n=3$, the most likely positions of particle are $x = \frac{a}{6}, \frac{3a}{6}, \frac{5a}{6}$.

The variation of ~~par~~ probability densities $|\psi_1|^2$, $|\psi_2|^2$ and $|\psi_3|^2$ with x.



It is noted that quantum mechanical results vary drastically from the classical results.

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14. particle in a Rectangular Three - Dimensional infinite well.

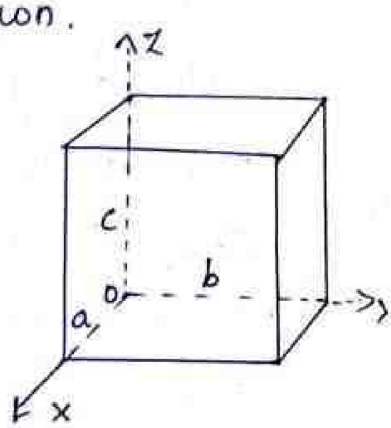
* Let a particle of mass m be in motion in a rectangular deep potential with sides of lengths a, b, c, parallel to the x, y and z - axes respectively.

* If there is no force acting on the particle inside the box, so that in the region.

$0 < x < a$

$0 < y < b$

$0 < z < c$



The potential energy $V(x, y, z) = 0$.

outside the box $V(x, y, z) = \infty$

Wave eqn of the particle

For the motion of the particle inside the box, the schrodinger time - independent wave eqn is,

$$-\frac{\hbar^2}{2m}\nabla^2\psi = E\psi \quad \ldots (1)$$

(or) $$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} + \frac{2m}{\hbar^2}E\psi = 0 \quad \ldots (2)$$

It assumed that the wave-function $\psi(x, y, z)$ is equal to the product of three functions X, Y, and Z each of which is a function of one variable only.

$$\psi(x, y, z) = X(x)\,Y(y)\,Z(z) \quad \ldots (3).$$

Substituting eqn (3) in eqn (2).

$$YZ\frac{\partial^2 X}{\partial x^2} + XZ\frac{\partial^2 Y}{\partial y^2} + XY\frac{\partial^2 Z}{\partial z^2} + \frac{2mE}{\hbar^2}XYZ = 0 \quad \ldots (4)$$

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Dividing eqn. (4) by XYZ,

$$\frac{1}{X}\frac{\partial^2 X}{\partial x^2} + \frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} + \frac{1}{Z}\frac{\partial^2 Z}{\partial z^2} + \frac{2mE}{\hbar^2} = 0 \quad \ldots (5).$$

In this eqn $\frac{2mE}{\hbar^2}$ is a constant for a particular value of the kinetic energy.

The kinetic energy E is expressed as the sum of the corresponding terms E_x, E_y and E_z.

$$E = E_x + E_y + E_z \quad \ldots (6).$$

From eqns (5) and (6),

$$\left[\frac{1}{X}\frac{\partial^2 X}{\partial x^2} + \frac{2mE_x}{\hbar^2}\right] + \left[\frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} + \frac{2mE_y}{\hbar^2}\right] + \left[\frac{1}{Z}\frac{\partial^2 Z}{\partial z^2} + \frac{2mE_z}{\hbar^2}\right] = 0$$

This equation gives three independent equations,

$$\frac{\partial^2 X}{\partial x^2} + \frac{2mE_x}{\hbar^2} X = 0 \quad \ldots (7)$$

$$\frac{\partial^2 Y}{\partial y^2} + \frac{2mE_y}{\hbar^2} Y = 0 \quad \ldots (8)$$

$$\frac{\partial^2 Z}{\partial z^2} + \frac{2mE_z}{\hbar^2} Z = 0 \quad \ldots (9).$$

The eqn (7) is the eqn for the one-dimensional case, the boundary condition applicable to the solution is,

$$X(0) = X(a) = 0.$$

So the eigen values of E_x are given by,

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$$E_x = \frac{\pi^2 \hbar^2}{2ma^2} n_x^2 \quad \ldots (10). \quad n_x = 1, 2, 3 \ldots$$

The corresponding normalized eigen functions are given by,

$$X(x) = \sqrt{\frac{2}{a}} \sin \frac{n_x \pi x}{a} \quad \ldots (11)$$

The solution for y and z are of the same form.

$$E_y = \frac{\pi^2 \hbar^2}{2mb^2} n_y^2 \quad \ldots (12)$$

$$Y(y) = \sqrt{\frac{2}{b}} \sin \frac{n_y \pi y}{b} \quad \ldots (13)$$

$$E_z = \frac{\pi^2 \hbar^2}{2mc^2} n_c^2 \quad \ldots (14)$$

$$Z(z) = \sqrt{\frac{2}{c}} \sin \frac{n_z \pi z}{c} \quad \ldots (15)$$

Eigen values of Energy :-

Substituting the expressions for E_x, E_y and E_z in eqn (6).

$$E_{n_x, n_y, n_z} = \frac{\pi^2 \hbar^2}{2m} \left[\frac{n_x^2}{a^2} + \frac{n_y^2}{b^2} + \frac{n_z^2}{c^2} \right] \quad \ldots (16)$$

These values are called the energy-levels of the particle.

Wave function :-

The total normalized wave function inside the box for the stationary states is given by,

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$$\psi_{n_x, n_y, n_z}(x,y,z) = X(x)\,Y(y)\,Z(z)$$

$$= \sqrt{\frac{8}{abc}} \sin\frac{n_x \pi x}{a} \cdot \sin\frac{n_y \pi y}{b} \cdot \sin\frac{n_z \pi z}{c} \quad \ldots (17)$$

* If the particle is confined in a cubical box in which $a = b = c = a$, the eigen-values of energy are given by,

$$E_{n_x n_y n_z} = \frac{\pi^2 \hbar^2}{2ma^2}\left(n_x^2 + n_y^2 + n_z^2\right) \quad \ldots (18)$$

In this case energy of the particle in the ground state is given by,

$$E_{111} = \frac{3\pi^2 \hbar^2}{2ma^2} \quad \ldots (19)$$

15. Correspondence principle

* In 1932 Niels Bohr proposed a correspondence principle.

* Bohr's correspondence principle bridges the gap between the classical mechanics and quantum mechanics.

* It removes the apparent discontinuity between the two.

Statement:

The principle states that for large quantum numbers quantum physics gives the same results as those of classical physics.

Proof

The velocity of an electron revolving round the

(36)

nucleus in an orbit of radius r is given by

$$v^2 = \frac{ke^2}{mr} \quad \dots (1)$$

Taking root on both sides,

$$\sqrt{v^2} = e\sqrt{\frac{ke^2}{mr}} = \frac{\sqrt{k}\sqrt{e^2}}{\sqrt{mr}}$$

$$v = e\sqrt{\frac{k}{mr}} = \frac{e}{\sqrt{4\pi\epsilon_0 mr}} \quad \dots (2)$$

$$r = \frac{n^2h^2}{4\pi^2 mkZe^2}$$

$$r = \frac{n^2h^2}{\frac{4\pi^2 m \times 1}{4\pi\epsilon_0} \times 1 \times e^2} = \frac{n^2h^2\epsilon_0}{\pi me^2} \quad \dots (3)$$

Putting $Z = 1$ for hydrogen and $k = \frac{1}{4\pi\epsilon_0}$

The frequency of revolution,

$$\nu = \frac{v}{2\pi r} \quad \dots (4)$$

Substituting for v we have,

$$\nu = \frac{1}{2\pi}\frac{e}{\sqrt{(4\pi\epsilon_0 mr)}\,r} = \frac{1}{2\pi}\frac{e}{(4\pi\epsilon_0 mr)^{1/2} r^1}$$

$$= \frac{1}{2\pi}\frac{e}{(4\pi\epsilon_0 m)^{1/2} r^{1/2} e^1}$$

$$= \frac{1}{2\pi}\frac{e}{(4\pi\epsilon_0 m)^{1/2} r^{3/2}}$$

Substituting for r, we have

$$= \frac{1}{2\pi} \cdot \frac{e}{(4\pi\epsilon_0 m)^{1/2}\left(\frac{n^2h^2\epsilon_0}{\pi me^2}\right)^{3/2}}$$

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(37)

$$\nu = \frac{me^4}{4\varepsilon_0^2 h^3} \cdot \frac{1}{n^3} \quad \ldots (5)$$

According to Bohr's theory of the hydrogen atom,

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$= \frac{me^4}{8\varepsilon_0^2 h^3}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\nu = \frac{c}{\lambda} = \frac{me^4}{8\varepsilon_0^2 h^3}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \ldots (6)$$

when the quantum number involved are large $n_1 = n$ and $n_2 = n+1$ where $n >> 1$,

$$\nu = \frac{me^4}{8\varepsilon_0^2 h^3}\left(\frac{1}{n^2} - \frac{1}{(n+1)^2}\right)$$

$$= \frac{me^4}{8\varepsilon_0^2 h^3}\left[\frac{(n+1)^2 - n^2}{n^2 (n+1)^2}\right]$$

$$= \frac{me^4}{8\varepsilon_0^2 h^2}\left[\frac{n^2 + 1^2 + 2n - n^2}{n^2 (n+1)^2}\right]$$

$$= \frac{me^4}{8\varepsilon_0^2 h^3}\left[\frac{2n+1}{n^2 (n+1)^2}\right]$$

As $n>>1$, so neglecting 1 as compared to n and 2n,

$$\nu = \frac{me^4}{8\varepsilon_0^2 h^3} \cdot \frac{2}{n^3} \quad \ldots (7)$$

$$\nu = \frac{me^4}{4\varepsilon_0^2 h^3} \quad \ldots (8)$$

comparing equations (4) and (5), we find that

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(38)

the classical orbital frequency and frequency of radiation emitted as calculated on the basis of quantum theory have the same value.

* The greater the quantum number, the closer quantum physics approaches classical physics.

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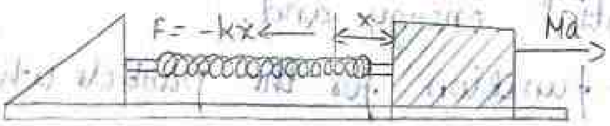
(1)

Unit - V
Applied Quantum Mechanics

1. Harmonic oscillator (Qualitative)

Definition:-

A particle undergoing simple harmonic motion is called a harmonic oscillator.



Example:-

If applied force moves the particle through x, then restoring force F is given by,

$$F \propto -x$$

$$F = -kx \quad \dots (1)$$

The potential energy of the oscillator is

$$V = -\int F\,dx$$

$$V = k\int x\,dx = \frac{1}{2}kx^2$$

$$\boxed{V = \frac{1}{2}kx^2} \quad \dots (2)$$

where k is force constant.

In harmonic oscillator, angular frequency is given by.

$$\omega = \sqrt{\frac{k}{m}}$$

Squaring on both sides.

$$\omega^2 = \left(\sqrt{\frac{k}{m}}\right)^2$$

$$\omega^2 = \frac{k}{m}, \quad k = m\omega^2$$

where, m = mass of the particle.

Substituting k in eqn (1), we have,

$$V = \frac{1}{2}m\omega^2 x^2 \quad \dots (3)$$

Wave equation for the oscillator:-

The time-independent Schrodinger wave equation for linear motion of a particle along the x-axis is,

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E-V)\psi = 0 \quad \cdots (A)$$

where, E - Total energy of the particle,
V - potential energy and.
ψ - wave-function for the particle which is function of x alone.

Substituting for V in equation (A)

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}\left(E - \frac{1}{2}m\omega^2x^2\right)\psi = 0 \quad \cdots (5)$$

$$\frac{d^2\psi}{dx^2} + \frac{2mE}{\hbar^2} - \frac{2m}{\hbar^2} \times \frac{1}{2} m\omega^2x^2\psi = 0$$

or $$\frac{d^2\psi}{dx^2} + \left(\frac{2mE}{\hbar^2} - \frac{m^2\omega^2}{\hbar^2}x^2\right)\psi = 0 \quad \cdots (6)$$

This is Schrodinger wave equation for the oscillator.

Simplification of the wave equation:-

To simplify eqn. (6) a dimensionless independent variable y is introduced. It is related to x by the equation

$$y = ax \quad \cdots (7)$$

$$\therefore x = \frac{y}{a}, \quad \text{where } a = \sqrt{\frac{m\omega}{\hbar}}$$

$$\frac{d\psi}{dx} = \frac{d\psi}{dy}\frac{dy}{dx} = \frac{d\psi}{dy}a$$

($y = ax$
$dy = a\,dx$
$dy/dx = a$)

Differentiating,

$$\frac{d^2\psi}{dx^2} = \frac{d^2\psi}{dy^2}\frac{d^2y}{dx^2}$$

$$\frac{d^2\psi}{dx^2} = \frac{d^2\psi}{dy^2} a^2 \qquad \frac{d^2y}{dx^2} = a^2.$$

$$\frac{d^2\psi}{dx^2} = a^2 \frac{d^2\psi}{dy^2} \quad \ldots (8)$$

$$a^2 \frac{d^2y}{dy^2} + \left(\frac{2mE}{\hbar^2} - a^4 \frac{y^2}{a^2}\right)\psi = 0 \qquad \left(\because x = \frac{y}{a}\right)$$

$$a = \sqrt{\frac{m\omega}{\hbar^2}}$$

$$a^2 \frac{d^2\psi}{dy^2} + \left(\frac{2mE}{\hbar} - a^2y^2\right)\psi = 0. \qquad a^2 = \frac{m\omega}{\hbar^2}$$

Dividing through out by a^2, we have,

$$a^2 = \frac{m^2\omega^2}{\hbar^2}$$

$$\frac{d^2\psi}{dy^2} + \left(\frac{2mE}{a^2\hbar^2} - y^2\right)\psi = 0 \quad \ldots (9) \qquad a^4 = \frac{m^2\omega^2}{\hbar^2}$$

Substituting for a^2

$$\frac{d^2\psi}{dy^2} + \left(\frac{2mE}{\frac{m\omega}{\hbar}\cdot\hbar^2} - y^2\right)\psi = 0 \quad \ldots (10)$$

$$\frac{d^2\psi}{dy^2} + \left(\frac{2E}{\hbar\omega} - y^2\right)\psi = 0$$

or
$$\frac{d^2\psi}{dy^2} + (\lambda - y^2)\psi = 0 \quad \ldots (11)$$

$$\lambda = \frac{2E}{\hbar\omega}$$

Eigen values of the total energy E_n :-

The wave eqn for the oscillator is satisfied only for discrete values of total energies given by.

$$\frac{2E}{\hbar\omega} = (2n+1)$$

(or)
$$E_n = \frac{1}{2}(2n+1)\hbar\omega$$

$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega \quad (12)$$

$$\hbar = \frac{h}{2\pi} \text{ and } \omega = 2\pi\nu.$$

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$$E_n = \left(n + \frac{1}{2}\right) h\nu \quad \cdots \quad (13)$$

$n = 0, 1, 2 \ldots$, and ν is the frequency of the classical harmonic oscillator, given by.

$$\nu = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \qquad \left(\because \omega = \sqrt{\frac{k}{m}}\right)$$

For eqn (13), we get the following conclusions.

The lowest energy of the oscillator is obtained by putting $n = 0$ in eqns (12) and (13) it is,

$$E_0 = \frac{1}{2}\hbar\omega = \frac{1}{2}h\nu \quad \cdots \quad (14).$$

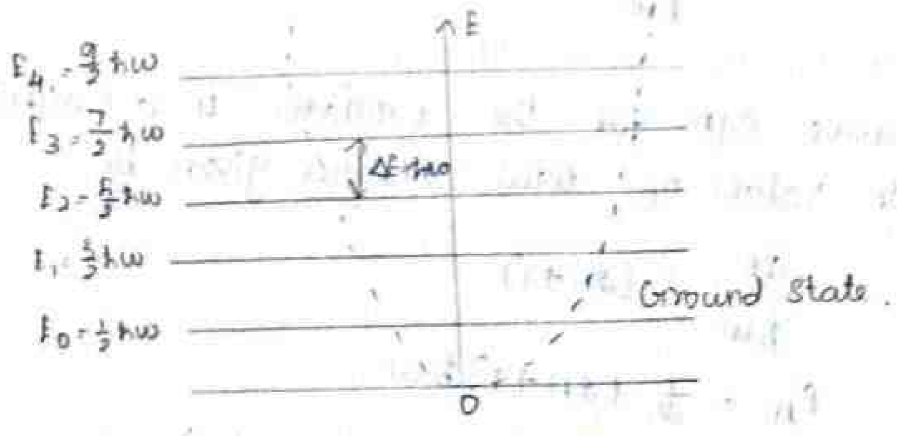
This is called the ground state energy or the zero point vibrational energy of the harmonic oscillator.

$$E_n = (2n+1) E_0 \quad \cdots \quad (15)$$

where $n = 0, 1, 2, 3 \ldots$

The eigen-values of the total energy depend only on one quantum number n.

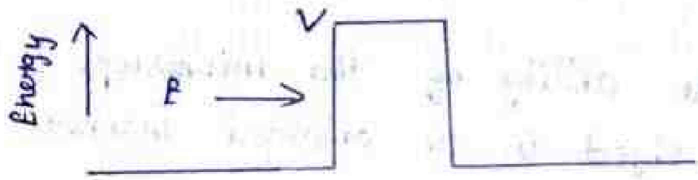
Therefore all the energy-levels of the oscillator are non-degenerate

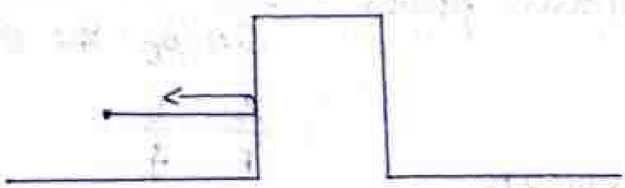


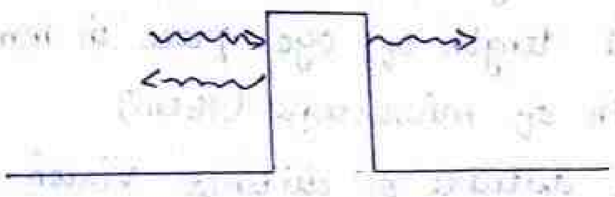
the potential energy $V = \frac{1}{2}kx^2$.

2. Barrier Penetration And Quantum Tunneling (Qualitative)

* According to classical ideas, a particle striking a hard wall has no change of leaking through it.
* We know that when an electromagnetic wave strikes at the interface of two media.
* A particle with energy $E < V$ approaching potential barrier of height V.
* An electron of total energy E approaches the barrier from the left. E is less tan V.
* For the particle to overcome the potential barrier, it must have an energy equal to or greater than V.







* It is noted that the electron tunneled through the potential barrier and hence in quantum mechanics, this phenomenon is called tunneling.
* The transmission of electrons through the barrier is known as barrier penetration.

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Terminology related to microscope:-

a) Microscope:-

A microscope is an instrument which is used to view the magnified image of a smaller object which cannot be clearly seen with a naked eye.

b) optical microscope:-

It is a microscope which uses light radiation to illuminate the object.

c) Resolving power:-

It is the ability of the microscope to show two closer objects as separated ones.

d) Magnification power:-

It is the ability of the microscope to show the image of an object in an enlarged manner.

$$\text{Magnification power} = \frac{\text{Size of the image}}{\text{Size of the object}}$$

$$= \frac{\Delta}{F} \cdot \frac{D}{f}$$

In an optical microscope:-

$F \rightarrow$ Focal length of objective lens in mm.

$f \rightarrow$ Focal length of eye piece in mm.

$\Delta \rightarrow$ Length of microscope (16cm)

$D \rightarrow$ least distance of distinct vision (25 cm)

e) Depth of focus:-

It is defined as the ability of the objective of microscope to produce a sharp focused image when the surface of the object is not truly plane.

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Electron Microscope:-

Definition:-

It is a microscope with uses electron beam to illuminate a specimen and it produces an enlarged image of the specimen.

principle:-

Higher magnification as well as resolving power can be obtained by utilizing waves of shorter wavelength (λ)

Electron microscope uses electron waves whose wavelength is given by the formula $\lambda = \frac{12.25}{\sqrt{V}}$

For $V = 10,000$ V, $\lambda = 0.1225$ Å which is extremely short.

Electron microscopes giving magnification more than 2,00,000 x are common in Science & Technology Medical Research Laboratories.

An electron microscope consists of the following essential parts.

Electron Gun:- Its function is to provide a narrow beam of electrons of uniform velocity.

Electrostatic and magnetic lenses:-

Their function is to refract and properly focus the electron beam.

Fluorescent Screen or Photographic plate:-

They are used to receive the highly magnified image of the extremely small object being studied.

Types of Electron Microscopes:-

There are four types of electron microscope.

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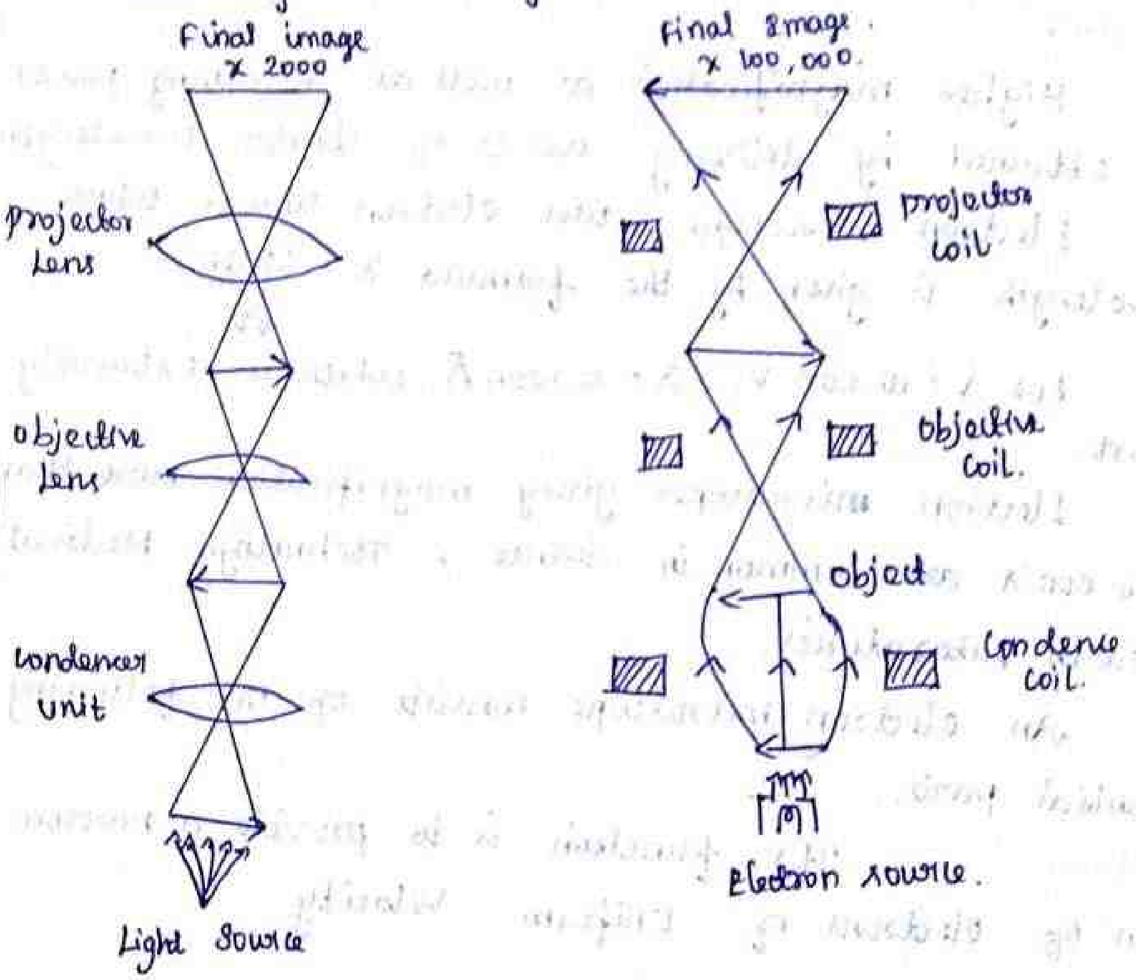
They are,

1. Transmission Electron Microscope (TEM)
2. Scanning Electron Microscope (SEM)
3. Scanning Transmission Electron Microscope (STEM).
4. Scanning Tunneling Microscope (STM)



3. Scanning Tunneling Microscope (STM).

* A Scanning ~~lens~~ tunneling microscope, or STM, is a type of electron microscope. It is commonly used in fundamental and industrial research.

* It is an instrument used for imaging surface at the atomic level

* Due to its high resolution, individual atoms within materials are routinely imaged and manipulated.

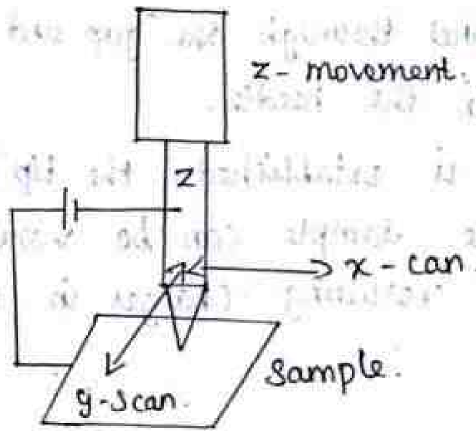
Principle:-

* It is based on the concept of quantum mechanical tunneling of electrons.

* In this technique, a sharp narrow conducting needle or tip is brought very near to the surface to be examined

* A small voltage difference about 1V is applied between the tip and surface of the material.

* This allows electrons to tunnel through the vacuum between them and results in tunneling current.



Construction:-

The components of STM include.

* Scanning needle tip.
* piezoelectric controlled height & and surface scanner.
* coarse sample to tip control.
* vibration isolation system and.
* computer

* Needle tip for scanning the sample surface. It is often made of tungsten.

* piezoelectric tube is provided with tip and electrodes. It is capable of moving x, y, z. directions.

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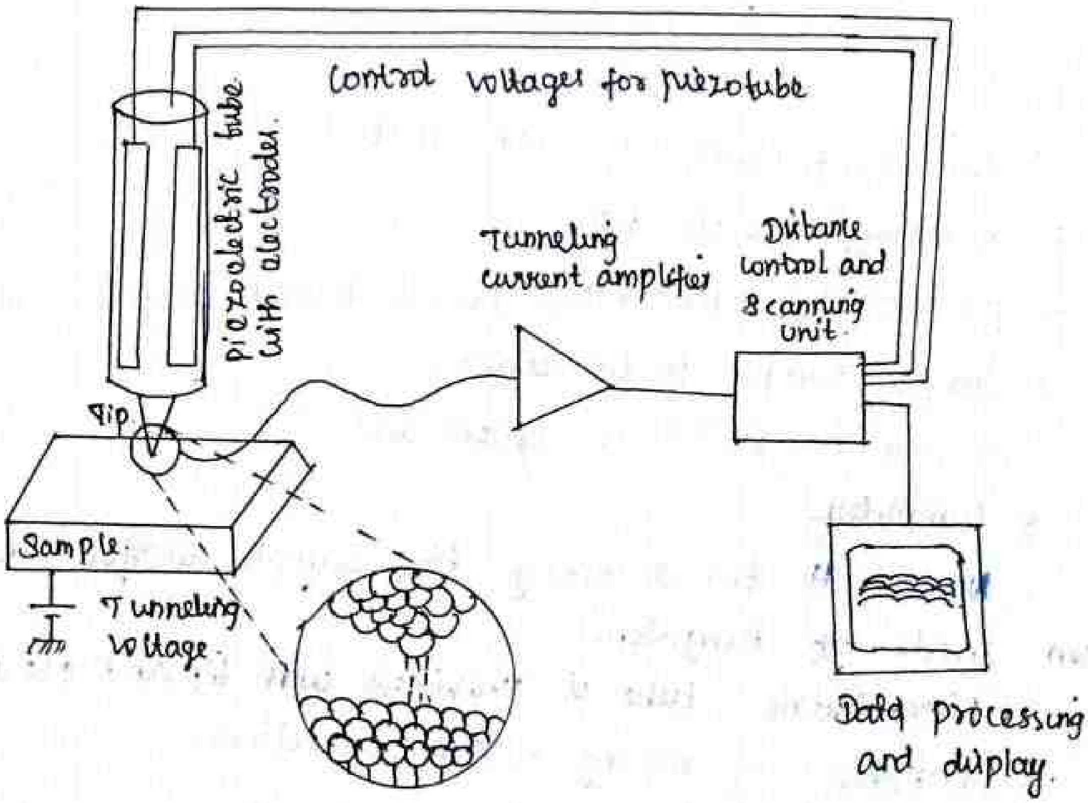
* Coarse sample to tip control is used to bring the tip close to the sample.
* Vibration isolation system, It prevents any vibration or sound in the system.

Working :-

* The sharp metal needle is brought close to the surface to be imaged. The distance is of the order of a few angstroms.
* A bias voltage is applied between the sample and the tip. The electrons can tunnel through the gap and set up a small, "tunneling current" in the needle.
* Once tunneling is established, the tip's bias and position with respect to the sample can be varied and data are obtained from the resulting changes in current.



Scanning :

* If the tip is moved across the Sample in the x-y plane, the changes in Surface height and density of states causes changes in tip current.

* These changes are mapped in images to present the Surface morphology.

* This change in current with respect to position can be measured itself.

* these two modes are called constant height mode and constant current mode, respectively.

Advantages of STM :-

* STMs are helpful because they can give researchers a three dimensional profile of a surface.

* For an STM, good resolution is 0.1 nm lateral resolution and 0.01 nm depth resolution.

* The high resolution of STMs enable researchers to examine surfaces at an atomic level.

* STM are also versatile. they can be used in ultra high vacuum, air, water and other liquids and gases.

* they will operate in temperatures as low as zero kelvin up to a few hundred degrees celsius.

Disadvantages of STM :-

* They are very few disadvantages of using a scanning tunneling microscope.

* STMs can be difficult to use effectively.

* There is a very specific technique that requires a lot of skill and precisions.

* A small vibration even a sound, can distrub the tip and the sample together.

* Even a single dust particle can damage the needle.
* STMs are highly specialized equipment that is fragile and expensive.

Applications of STM:-

* It is a powerful tool used in many research fields and industries to obtain atomic scale sample imaging and magnification.
* It is used to analyze the electronic structure of the active sites at catalyst surfaces.
* STM is used in the study of structure, growth morphology, electronic structure of surface, thin films and nano structures.

4. Resonant Diode:-

* It is a device that that has two tunneling junctions.
* Its I-V characteristic shows negative differential resistance characteristic

Definition:-

* A resonant tunneling diode (RTD) is a diode with resonant tunneling structure.
* The electrons can tunnel through some resonant states at certain energy levels.

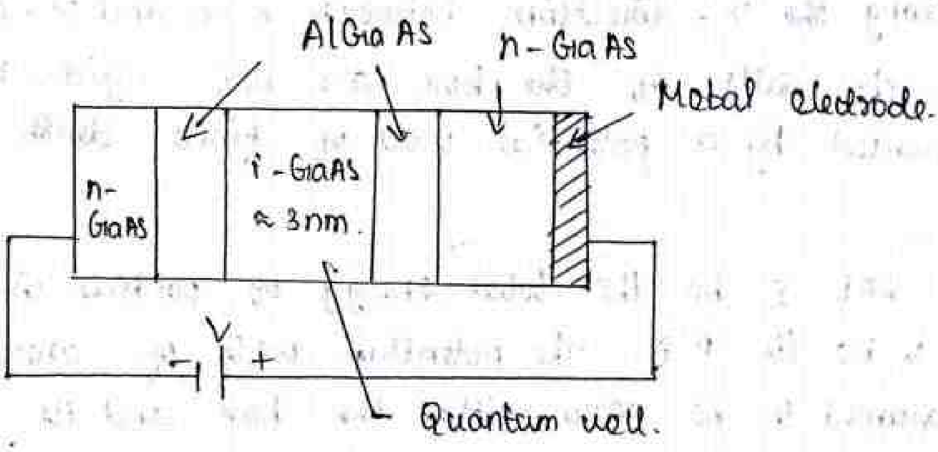
Principle:-

* When electron incident with energy equal to energy level of a potential well of thin barrier, then the tunneling reaches its maximum value.
* This is known as resonant tunneling.

Structure of RTD

A typical resonant tunneling diode structure.

is made by using n-type GaAs for the regions to the left and right of both barriers. 13



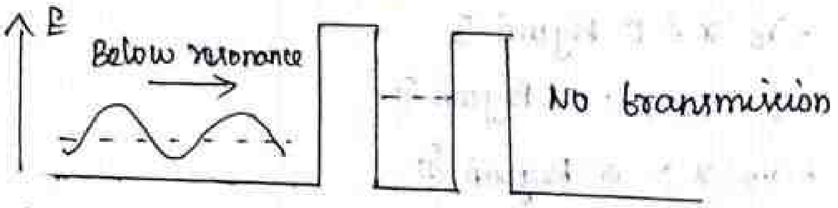
working:

Tunneling control:-

Tunneling is controlled by applying a bias voltage across the device.

without applied bias.

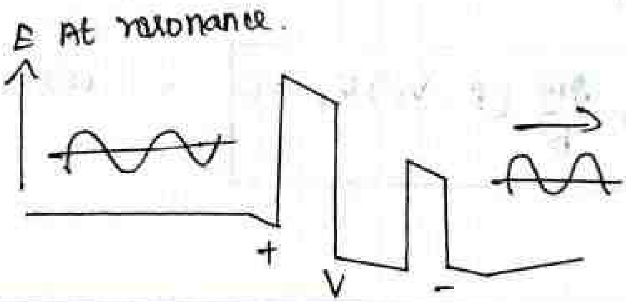
For the case of no applied bias, the energy band diagram is shown in fig.



This energy matching and hence resonant tunneling could be achieved by biasing the potential barrier.

with applied bias:

when voltage is applied, the band diagram shifts and if the voltage is varied until the quantized discrete energy level.



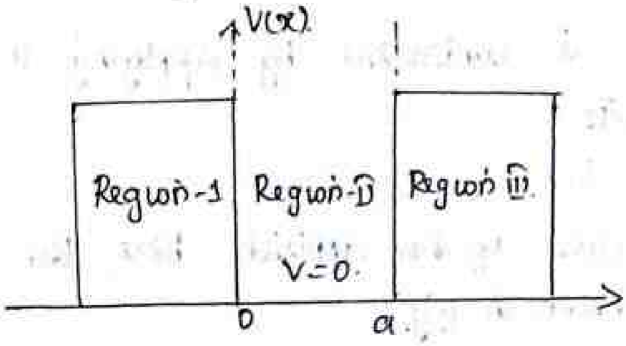
5. particle in a finite potential wells (qualitative)

* consider a particle of mass m moving with velocity v along the x-direction between $x=0$ and $x=a$.

* the walls of the box are not rigid. Hence it is represented by a potential well of finite depth.

Step-1 :-

Let E be the total energy of particle inside the box and v be its P.E. The potential well of energy which is assumed to be zero within the box and its value outside the box is finite say V_0 and $V_0 > E$.



$V(x) = V_0 \quad x \leq 0$ Region I

$V(x) = 0 \quad 0 < x < a$ Region II

and $V(x) = V_0 \quad x \geq a$ Region III

$$\boxed{\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E-V)\psi = 0} \quad \cdots (1)$$

Step-II :-

consider the three regions I, II, III separately and let $\psi_{I}, \psi_{II}, \psi_{III}$ be the wave functions in them respectively.

we have region-I

$$\boxed{\frac{d^2\psi_1}{dx^2} + \frac{2m}{\hbar^2}(E-V_0)\psi_1 = 0} \quad \cdots (2)$$

For region II.

$$\boxed{\frac{d^2\psi_{II}}{dx^2} + \frac{2mE}{\hbar^2}\psi_{II} = 0} \quad \ldots (3) \qquad (\because V=0)$$

and for region III.

$$\boxed{\frac{d^2\psi_{III}}{dx^2} + \frac{2m}{\hbar^2}(E - V_0)\psi_{III} = 0} \quad \ldots (4)$$

Let $\frac{2mE}{\hbar^2} = k^2$ and $\frac{2m(E-V_0)}{\hbar^2} = -k'^2 \quad \ldots (5)$

Then the eqn in the three regions is written as.

$$\boxed{\begin{aligned} &\frac{d^2\psi_{I}}{dx^2} - k'^2\psi_{I} = 0. \\ &\frac{d^2\psi_{II}}{dx^2} + k^2\psi_{II} = 0. \\ &\frac{d^2\psi_{III}}{dx^2} + -k'^2\psi_{III} = 0 \end{aligned}} \quad \ldots (6)$$

Step - III :- The solutions of these equations are of the form:

$$\psi_{I} = Ae^{k'x} + Be^{-k'x} \quad \text{for } x<0.$$

$$\psi_{II} = P.\, e^{ikx} + Q.\, e^{-ikx} \quad \text{for } 0<x<a.$$

$$\psi_{III} = C.\, e^{k'x} + De^{-k'x} \quad \text{for } x>a.$$

Step - IV: As $x \to \pm\infty$, ψ should not become infinite. Hence. $B=0$ and $C=0$.

$$\boxed{\begin{aligned} &\psi_{I} = Ae^{k'x} \\ &\psi_{II} = P.\, e^{ikx} + Q.\, e^{-ikx} \\ &\psi_{III} = D.\, e^{-k'x}. \end{aligned}}$$

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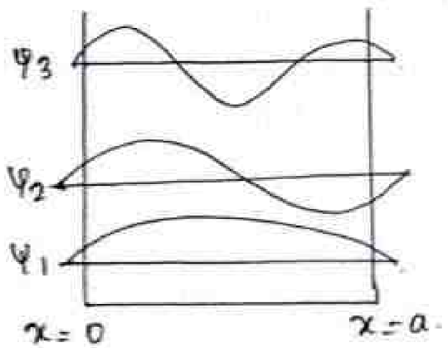
Step V :- The constants A, P, Q and D can be determined by applying the boundary conditions.

$$\psi_I(0) = \psi_{II}(0).$$

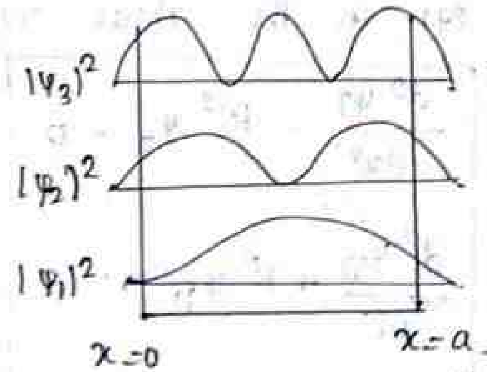
$$\left[\frac{d\psi_I}{dx}\right]_{x=0} = \left[\frac{d\psi_{II}}{dx}\right]_{x=0}$$

$$\psi_{II}(a) = \psi_{III}(a)$$

$$\left[\frac{d\psi_{II}}{dx}\right]_{x=a} = \left[\frac{d\psi_{III}}{dx}\right]_{x=a} \quad \text{--- (8)}$$



a) wave functions



b) probability densities inside non-rigid box.

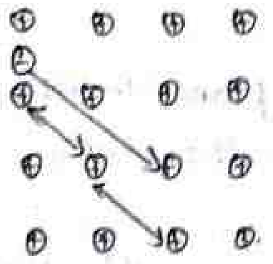
* The eigen functions are similar in appearance to those of infinite well except that they extend a little outside the box.
* this shows penetration of the particle into the classically forbidden region.

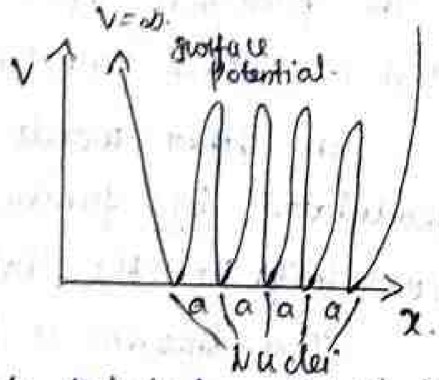
Band theory of solids :-

A solution to this problem was given by band theory of solids and is called zone theory.

postulates :-

Inside a real crystal, the electrons (-ve charge) move through periodic arrangement of positively charged holes.

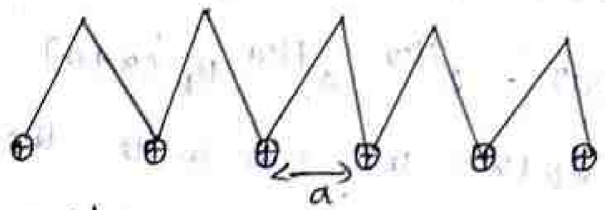




Shows one dimensional periodic potential distribution for a crystal.

6. BLOCH'S Theorem for particles in a periodic potential:-

The motion of electron inside the lattice is not free as expected, but the electron experiences a periodic potential variation.



Bloch theorem:-

It is a mathematical statement regarding the form of one electron wave function for a perfectly periodic potential.

Statement:-

If an electron in a linear lattice of lattice constant 'a' characterized by potential function $V(x) = V(x+a)$ satisfies the schrodinger equation.

$$\frac{d^2\psi(x)}{dx^2} + \frac{2m}{\hbar^2}(E - V(x))\psi(x) = 0 \quad \cdots (1)$$

$$\psi(x) = u_k(x)e^{ikx} \quad \cdots (2)$$

$$u_k(x) = u_k(x+a) \quad \cdots (3)$$

Here $u_k(x)$ is also periodic with lattice periodicity.

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The potential V(x) is periodic as $V(x) = V(x+a)$ where a is a lattice constant.

In other words the solutions are plane waves modulated by function $u_k(x)$ which has the same periodicity as the lattice.

This theorem is known as Bloch Theorem. The functions of the type (2) are called Bloch functions.

Proof:-

If eqn (1) has the solutions with the property of equation (2) we can write the property of the Bloch functions i.e, equation (3) as.

$$\psi(x+a) = e^{ik(x+a)} u_k(x+a)$$

(or) $$\psi(x+a) = e^{ikx} . e^{ika} u_k(x+a)$$

Since $u_k(x+a) = u_k(x)$, we can write the above eqn as,

$$\psi(x+a) = e^{ikx} . e^{ika} . u_k(x) \quad \ldots (4)$$

Since $\psi(x) = e^{ikx} u_k(x)$, we can write the above eqn as

$$\psi(x+a) = e^{ika} . \psi(x) \quad \ldots (5)$$

(or) $$\psi(x+a) = Q\psi(x) \quad \ldots (6)$$

$$Q = e^{ika}$$

If $\psi(x)$ is a single-valued function, then we can write $\psi(x) = \psi(x+a)$. Thus Bloch theorem is proved.

If the potential is a function of 'x' and 'a', then the wave function is also a function of 'x' and 'a'

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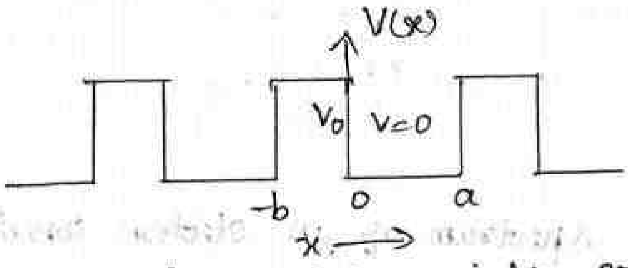
7. Basis of kronic penny model:-

The essential feature of the behaviour of electronic potential is studied by considering a periodic rectangular well structure in one dimension.

It was first discussed by Kronig and Penny in the year 1931.



One dimensional periodic potential.

The one dimensional schrodinger wave equations for two regions are written as.

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}[E-0]\psi = 0 \quad \text{for } 0 < x < a \quad \ldots (1)$$

(or) $$\frac{d^2\psi}{dx^2} + \alpha^2\psi = 0. \quad \ldots (2) \quad \left(\because \alpha^2 = \frac{2mE}{\hbar^2}\right)$$

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E-V_0)\psi = 0 \quad \text{for } -b < x < 0 \quad \ldots (3)$$

(or) $$\frac{d^2\psi}{dx^2} - \beta^2\psi = 0 \quad \ldots (4) \quad \left(\because \beta^2 = \frac{2m}{\hbar}(V_0-E)\right)$$

$$\psi = e^{-ikx}\, U_k(x) \quad \ldots (5)$$

Differentiating eqn (5) and substituting in eqn (2) and (4)

$$\frac{P \sin \alpha a}{\alpha a} + \cos \alpha a = \cos ka \quad \ldots (6)$$

$$\alpha = \sqrt{2mE}/\hbar \quad \text{and } P = \frac{mV_0 ba}{\hbar^2}$$

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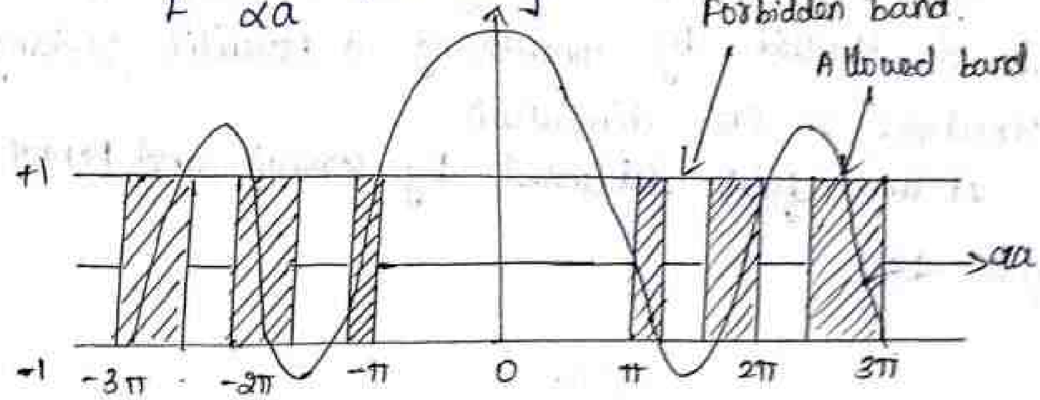
The eqn (6) is analysed by drawing a plot between αa and $\left[\frac{P \sin \alpha a}{\alpha a} + \cos \alpha a\right]$,



A plot of αa versus $\left(\frac{P \sin \alpha a}{\alpha a} + \cos \alpha a\right)$

From the graph,

The energy spectrum of an electron consists of a large number of allowed and forbidden energy bands.

This is because the first term of eqn $\frac{P \sin \alpha a}{\alpha a}$ decreases with increase of αa.

In the other extreme case, when $P \to 0$.

$$\cos \alpha a = \cos ka$$

$$\alpha^2 = k^2$$

$$\frac{2mE}{\hbar^2} = k^2 \qquad \left(\because \hbar = \frac{h}{2\pi}\right)$$

$$E = \frac{\hbar k^2}{2mE}$$

$$E = \frac{\hbar^2 k^2}{2mE} \Rightarrow \boxed{\frac{h^2 k^2}{8\pi^2 m}}$$

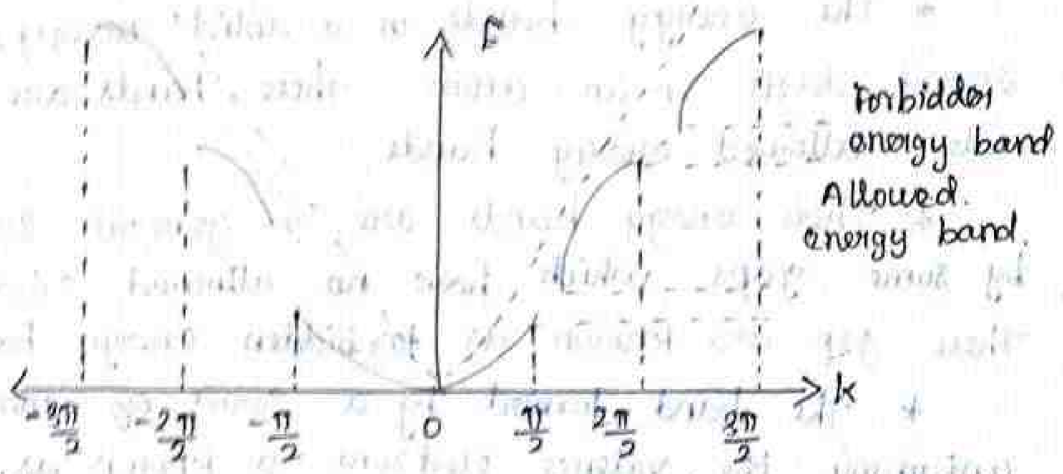
which corresponds to free electron model.

E - K curve:

The energy of the electron in the periodic lattice is given by,

$$\boxed{E = \frac{h^2 k^2}{8\pi m} \cdot k^2}$$

These zones are the allowed energy bands separated by forbidden energy bands.

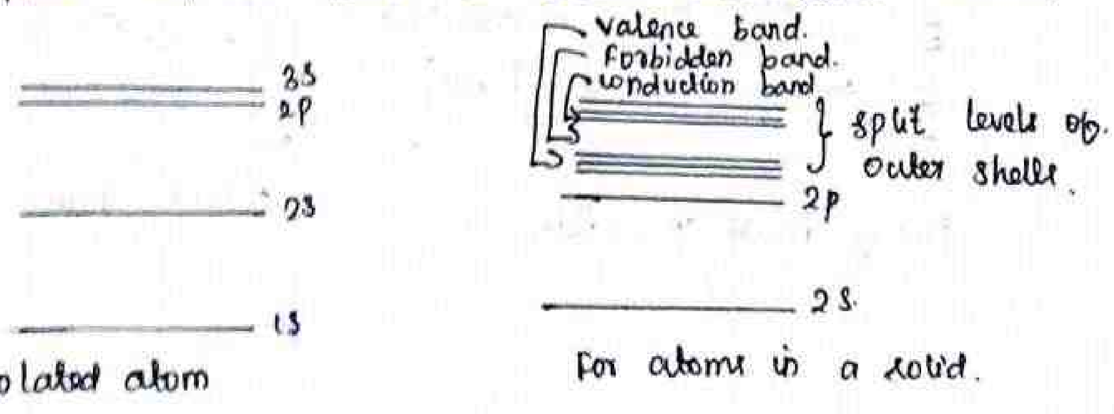


Plot of energy vs. wave vector in one dimensional Lattice

8. Origin of Energy Bands:-

A solid contains an enormous number of atoms closely together. In the case of a single isolated atom, there are discrete energy levels 1S, 2S, 2P, 3S... . These energy levels can be occupied by the electrons of the atom.

The lower completely filled band is valence band and upper unfilled band is called conduction band.



Definition :-

A set of such closely spaced energy levels is called an energy band.

Concept of valence band, conduction band and forbidden band.

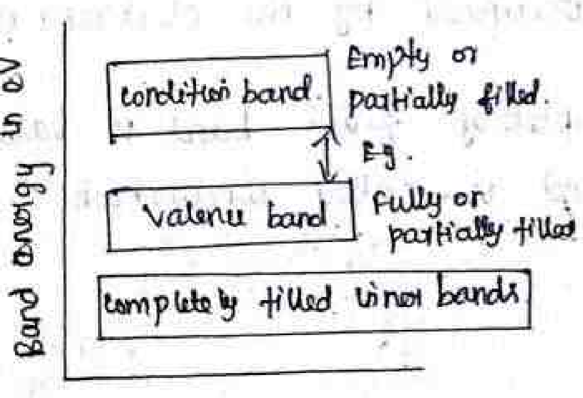
* The energy bands in a solid correspond to the energy levels in an atom. these bands are, therefore called allowed energy bands.

* These energy bands are, in general, separated by some gaps which have no allowed energy levels. These gap are known as forbidden energy bands.

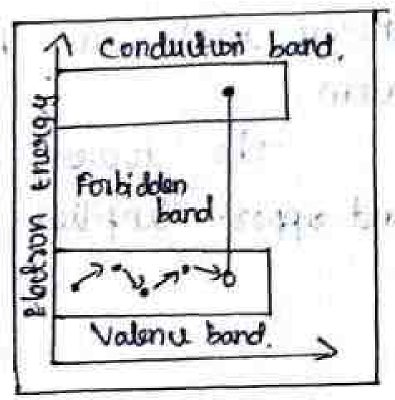
* The band formed by a series of energy level containing the valence electrons is known as valence band.

* The energy gap between the valence band and conduction band is called the forbidden energy gap or forbidden band.

When an electron in the valence band absorbs enough energy.



Energy bands in solids.



valence band.

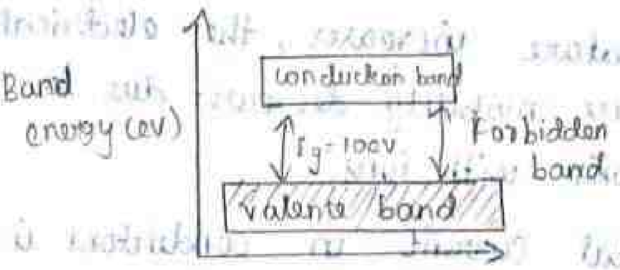
22.

classification of metals, semiconductors and insulators:-

On the basis of width of forbidden gap valence and conduction band the solids are classified into insulators, semiconductors and conductors.

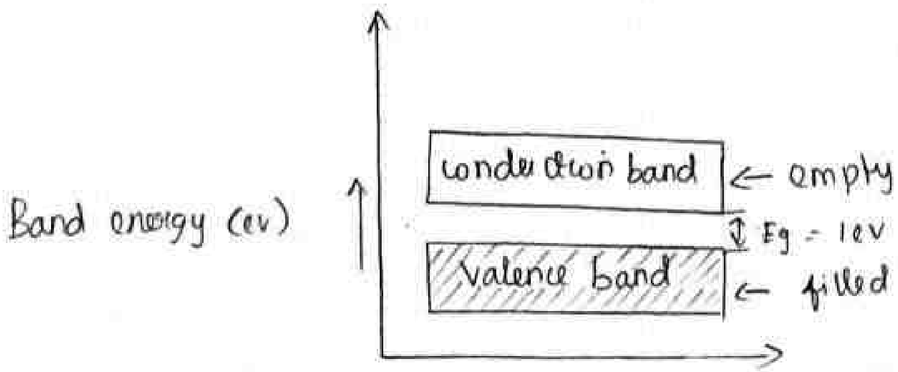
Insulators:-

* The band structure of insulators is as shown fig.
* The energy gap between condition band and valence band is very high and is about 10 eV.
* The conduction band is completely vacant and valence band is completely filled.



Semiconductors:-

* The band structure of semiconductors is as shown in fig.
* The forbidden gap is very small. Germanium and silicon are the best examples of semiconductors.
* The energy gap between conduction band and valence band is very small. It is about 0.5 eV to 1eV.



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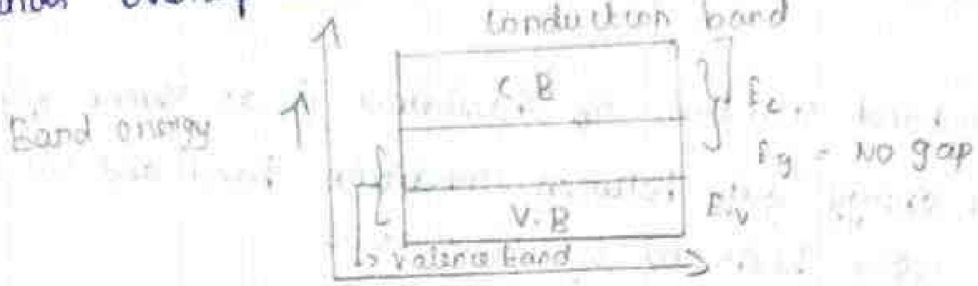
conductor:-

* The band structure of conductors is as shown in fig.
* There is no forbidden gap. both valence and conduction bands overlap each other.



* The electrons free to move within the conductor are responsible for electrical condition.
* As temperature increases, the electrical conduction decreases, because mobility decreases due to large number of excited collisions with ions.

The total current in conductors is due to only the flow of electrons.